Demographic Transition in Africa: the Polygyny and Fertility Nexus

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The World Bank CES - University Paris 1 Ottawa University

September 2011

Abstract

Sub-Saharan Africa is the only part of the world where the demographic transition is not about to be completed. Though mortality rate have declined since the 1960s, fertility remains still very high: around 5.1 children per women in average. What causes this delay in the demographic transition? What is the role of the particular familial structures such as polygyny, on fertility? To answer these questions, this paper presents theoretical and empirical evidence on the impact of polygyny on fertility in Sub-Saharan Africa. The theoretical model establishes that polygyny induces higher fertility. Monogamous family settled in area where polygyny is common, are likely to have more children than they would have in the absence of polygyny. Empirical estimations using DHS data confirm this result. They indicate that even if polygyny is likely to lower fertility at the individual level, the overall positive impact dominates. Indeed polygyny increases the incidence of marriage and decreases the age at which people first marry. Moreover, average fertility rates happen to be substantially higher in areas where polygyny is more frequent, even for women in monogamous family. Therefore, polygyny accounts for up to 0.7 point in fertility rate in some regions of our sample.

Keywords: Demography, Population economics, Fertility, Polygyny, Sub-Saharan Africa

JEL Classification: J12, J13

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Résumé

Le continent africain a atteint en 2009 le milliard d'habitants. La plupart de la croissance démographique en Afrique est due à l'Afrique subsaharienne (ASS). En effet, le taux de croissance démographique des pays arabes de l'Afrique est inférieur à 2% par an et leur taux de fécondité est d'environ 2,5 enfants par femme. La transition démographique dans ces pays a commencé au début des années 1970 et a évolué à un rythme rapide. La plupart des pays d'Afrique Subsaharienne n'ont pas vraiment commencé leur transition démographique ou connaissent une transition démographique très lente. Le taux de fécondité moyen de l'Afrique subsaharienne est d'environ 5,1 enfants par femme en 2009, illustrant un taux annuel moyen de croissance démographique de 2,5% pendant toutes les années 2000. Malgré les progrès réalisés en matière de soins de santé et dans l'éducation, à savoir dans l'enseignement primaire et secondaire, depuis l'adoption des Objectifs du Millénaire pour le développement au début des années 1990, le taux de croissance de la population est encore élevé.

Le but de cet article est d'analyser le rôle et les effets d'une caractéristique culturelle forte des sociétés africaines, à savoir la polygamie, sur les tendances démographiques des pays d'Afrique subsaharienne. Nous tentons d'expliquer l'impact de la structure familiale sur la transition démographique de l'Afrique subsaharienne. Les progrès dans l'éducation, en particulier des femmes, et la réduction de la mortalité infantile sont considérés comme les principaux moteurs de la transition démographique. Dans le schéma classique, la transition démographique commence avec la diminution du taux de mortalité, ce qui donne un régime de taux de croissance démographique élevé, ensuite le taux de fécondité diminue pour conduire le taux de croissance démographique à un nouveau régime stationnaire caractérisé par un faible niveau de fécondité et de taux de mortalité (voir Chesnais [1992] pour une analyse complète).

Les pays d'Afrique subsaharienne sont caractérisés par une persistance d'un régime de taux de fécondité élevé bien qu'il est baissé dans certains pays. Pour comprendre la fécondité des pays d'Afrique subsaharienne, il est nécessaire de tenir compte de variables qui sont intrinsèquement liées aux comportements de fécondité. La structure familiale est un candidat direct pour expliquer les comportements de fécondité. La polygamie a été suspectée de jouer un rôle clé sur la fertilité très tôt par les démographes. Muhsam (1956) et Dorjahn (1959) ont essayé d'établir une évaluation du rôle de la polygamie dans la fertilité

Dans ce papier, nous établissons, dans un modèle théorique de réseau, que la polygynie augmente le taux de fécondité dans une société. Tout d'abord, nous montrons que le nombre total de mariages est plus élevé dans une culture polygame que dans une culture monogame. Pison (1986) constate que la structure polygame peut augmenter la nuptialité dans une société, ce qui augmente le nombre de naissance et donc le taux de croissance de la population. Deuxièmement, bien que la femme dans une relation monogame ait plus d'enfants que la femme dans une relation polygame, nous montrons que la polygynie induit un effet positif de contagion sur la fertilité. Les hommes polygames ont plus d'enfants que les hommes monogames mais, par un effet d'externalité, un individu monogame dans une culture polygame a plus d'enfants qu'un individu monogame dans une culture monogame.

Nous fournissons des preuves empiriques de l'impact de la polygamie sur la fécondité en Afrique subsaharienne. Nous utilisons des données des enquêtes démographiques sur douze pays ayant une législation différente en ce qui concerne la polygamie. Notre estimation indique qu'au niveau individuel la polygynie un impact négatif sur le taux de fécondité. Les femmes dans une structure polygame ont tendance à avoir un taux de fécondité inférieur et significatif à celui des femmes dans une famille monogame. Cependant, la polygynie augmente également la nuptialité et le taux de remariage après un divorce ou un veuvage. Elle diminue aussi l'âge du mariage. Au final, la fécondité moyenne est sensiblement plus élevée dans les régions où la polygamie est plus fréquente.

Nous utilisons la taille de la femme comme une variable instrumentale. La variable taille semble être un bon instrument étant corrélée positivement avec la polygynie et pas corrélée avec le taux de fécondité. Notre évaluation globale indique que la structure familiale joue un rôle important dans le comportement de fécondité et peut expliquer les modèles de transition démographique de l'Afrique. Notre estimation indique que la polygamie compte jusqu'à 0,7 point du taux de fécondité dans certaines régions de notre échantillon.

1 Introduction

The African continent population has reached by 2009 the billion. Most of the demographic growth is now located in Sub-Saharan Africa (SSA). Indeed, population growth in the African Arabic countries is less than 2% per year and fertility rate is about 2.5 children per women. Demographic transition begun in these countries in the early 1970's and has been evolving at a fast pace. Many Sub-Saharan African countries however have not really started their demographic transition or are moving on a very low demographic transition path. The average total fertility rate in Sub-Saharan Africa was still around 5.1 children per women in 2009 and population grew at 2.5% a year in average during the 2000's. Despite of the progress made in health care and general education, since the adoption of the Millennium Development Goals in the early 1990, population growth remains very dynamic. The aim of this paper is to analyze the role and effects of a strong cultural feature of African societies, namely polygyny, on the demographic patterns of SSA countries. Progress in education, especially for women, and reduction of infant mortality are usually seen as the main drivers of the demographic transition. In the classical framework, demographic transition begins with a decline in mortality rates, yielding a regime of high population growth. Then fertility decreases, driving population growth to a new stationary regime characterized by a low fertility and mortality rate (see Chesnais [1992] for a complete analysis). The Sub-Saharan African countries are still featured by the persistence of high - although declining in some countries- fertility rates. As illustrated in Figure 1 below, while the infant mortality rate has declined steadily since the 1950's, the total fertility rate started to decline very slowly only in the late 1980's, yielding high population growth. These figures confirm that to understand SSA countries demographics one should take into account variables that are intrinsically related to fertility behaviors. Family structure appears to be a direct candidate. And polygyny has been suspected to play a key role on fertility very early by demographers. Muhsam (1956) and Dorjahn (1959) tried to establish an assessment of the role of polygyny in fertility[4]. In this paper, we establish in a theoretical network framework that polygyny increases fertility (proposition 7). The results follow from different implications of a polygynous familial structure. First, we show that the aggregate number of marriages is higher under a polygynous culture than under a monogamous culture (corollary 1 and proposition 2). This conclusion is consistent with Pison's (1986) findings that polygamous structure may increase the frequency of marriages in a society. This is likely to raise the number of child born and therefore to boost population growth. Second, although women in a monogamous relationship tend to have more children than women in a polygynous relationship (proposition 3), we show that polygyny induces a contagion effect on fertility. As polygynous man have more children than monogamous man, by an externality effect, a monogamous individual in a functioning polygynous culture compete by increasing the number of children he desires (corollary 3). In the second part of the paper, we provide empirical evidence of the impact of polygyny on fertility in Sub-Saharan Africa. We use for that purpose DHS data on twelve countries[5] (Nigeria, Senegal, Cote Ivory Coast, Cameroon, Rwanda, Tanzania,

[4]See Lardoux and Van de Walle (2003) for recent studies on specific Senegalese ethnicities and Borgerhoff Mulder (1989) for studies on Kipsigis women in Kenya.

[5]The Demographic and Health Survey (DHS) used in this study were carried out by : the Statistical and Health Services (Ghana), the Institut National de la Statistique (Cote d'Ivoire), the Institut National de la Statistique (Benin), the Ministère du Plan et de l'Aménagement du Territoire (Cameroon, 1991), the Bureau Central des Recensements et Etudes de Population (Cameroon, 1998), the Institut National de la Statistique (Cameroon, 2004), the Centre National de Recherches sur l'Environment (Madagascar,

Uganda, Zimbabwe, Benin, Ghana, Malawi and Madagascar) with different common and legal practices regarding polygyny. Our estimation indicates that at the individual level polygyny has a negative impact on fertility rate. That is women in a polygynous structure tend to have a significant lower fertility rate than women in monogamous family. However, polygyny appears also to increase both the nuptial rate and the rate of remarriage after a loss or a separation. It also decreases the age of marriage. Moreover, average fertility happens to be substantially higher in the areas where polygyny is more frequent. We use the height of women as an instrumental variable, it appears that at the cluster or regional level, polygyny and fertility are positively correlated. The height variable appears to be a good instrument correlated positively with polygyny and not correlated with the fertility rate. Our overall assessment indicates that family structure plays an important role in fertility behavior and may explain the patterns of the Africa's demographic transition. Polygyny account for up to 0.7 point in fertility rate in some regions from our sample. Our robustness check looks at the impact of variables characterizing behaviors or beliefs of both men and women at the local level on the fertility rate. Although correlated with both polygyny and fertility/nuptial those variables, which depict cultural and societal beliefs, do not explain the previous correlations. The paper is organized as follows: section (2) presents the related literature, linkages and motivations, section (3) displays the theoretical framework and results, section (4) provides the empirical findings, section (5) shows the robustness check. In section (6) we simulate the effects of polygyny at the macro level on population growth rate depending on our sample.

2 Demographic transition, Fertility and Polygyny

2.1 The classic drivers of the demographic transition

Demographic transition is seen as one of the powerful forces that induce the transition to modern economy. According to the Galor and Weil's (1999) and (2000) unified growth model, the period during the two phases of the demographic transition played a central role in setting the roots of modern growth. The demographic transition is usually described as two distinct phases. The mortality rates decline first, giving the start of the demographic transition. Then follows a long period of high population growth, as fertility rates remain high, which varies with countries' specificities[6]. The second phase of the demographic transition begins with the fertility decline. What factors drive the onset of the decline in fertility is still controversial. There is a vast theoretical, empirical and historical literature on the demographic transition[7]. For demographers, it is the decline of the mortality rate that causes ultimately the decline of the fertility rate (see Dyson, 2010). Among economists, Nerlove (1974) initiated a theory linking high mortality to high fertility[8].

1992), the Institut National de la Statistique (Madagscar, 1997, 2003, 2008), the National Statistical Office (Malawi), the Federal Office of Statistics (Nigeria, 1990), the National Population Commission (Nigeria, 1999, 2003, 2008), the Office National de la Population (Rwanda, 1992, 2000), the Ministry of Economics (Rwanda, 2005), the Ministère des Finances (Senegal, 1992, 1997), the SERDHA (Senegal, 1999), the Ministère de la Santé, CRDH (Senegal, 2005, 2006), the National Bureau of Statistics (Tanzania), the Bureau of Statistics (Uganda) and the Central Statistical Office (Zimbabwe). ICF Macro, an ICF International company, provided financial and technical assistance for the survey through the USAID-funded MEASURE DHS programme (http://www.measuredhs.com).

[6] France and United States are notable exception of this process.

[7] which has been surveyed by Galor (2005, 2010).

[8] See also Kalemli-Ozcan (2002).

However, the fact that in Western Europe the mortality decline started nearly a century prior to the decline in fertility casts some doubt in the causal relationship. Moreover, Doepke (2005), using mortality and fertility data from England during 1861–1951, found that in the absence of changes in other factors, the decline in child mortality during this time should have resulted in a rise in net fertility rates. Fernández-Villaverde (2001) met the same conclusion that declining mortality is probably not the main driver of fertility decline. As initially argued by Becker (1960), the rise in income could be seen as another driver of the fertility decline. Women participation rates increased with industrialization, driving up the opportunity cost of bearing children. This process explains the fall of the fertility rate. However, during the nineteen century, Western European countries that differed significantly in their income per capita experienced simultaneously the same demographic transition process. Moreover, the results presented by Murtin (2009)[9] show that income per worker was positively correlated with fertility rates, once mortality and education were controlled for. But, Jones and Tertilt (2006) documented a strong negative relationship between fertility and income at the individual level from US census data for women born between 1826 and 1960. They estimate an overall income elasticity of about -0.38 for the period. The other potential trigger of the fertility decline is the rise in human capital. Becker (1973; 1981) first pointed out the role of human capital in fertility choice within the household but this idea was theorized by Galor and Weil (1999, 2000) and Galor and Moav (2002). According to this theory, the Industrial Revolution increased the demand for human capital and female labor (Galor and Weil (1996)), both generating additional income that relaxed households' budget constraints and allow them increasing their investments in children's human capital. Then, increasing human capital returns due to technological change incited households to prefer quantity for quality of children (Becker and al. (1990), Tamura (1996), Galor and Weil (2000), De La Croix and Doepke (2003)), inducing a decline in fertility. Murtin (2009) provides evidence from a panel of countries during 1870—2000 that demonstrates that investment in education was indeed a dominating force in the decline in fertility. In particular, educational attainment has been negatively associated with fertility, accounting for income per worker and mortality rates. Also, Murphy (2009) finds, based on panel data from France during 1876–1896, that education attainment had a negative impact on fertility rates during France's demographic transition, accounting for income per capita, the gender literacy gap, and mortality rates . Moreover, quantitative evidence provided by Doepke (2004) suggests that educational policies aimed at promoting human capital formation played an important role in the demographic transition in England.

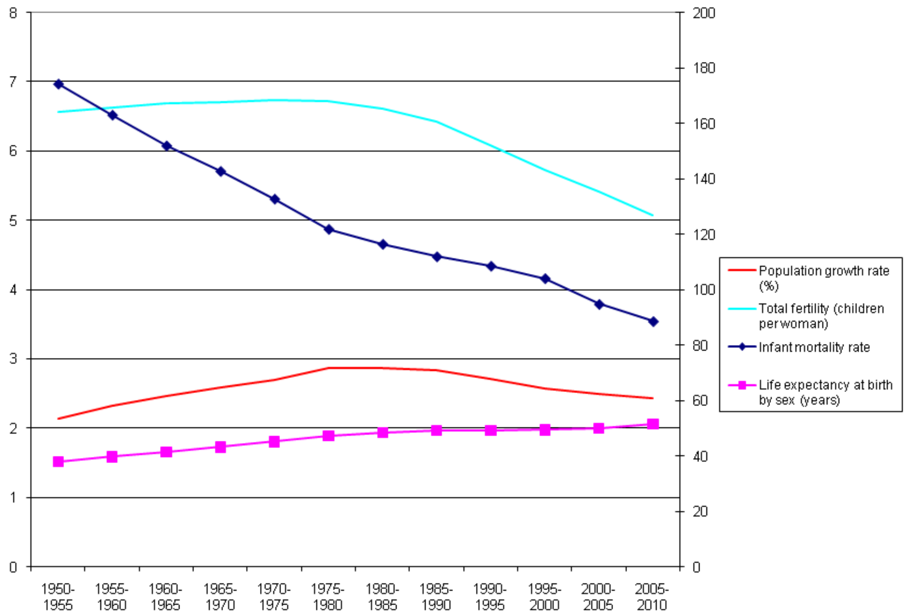
2.2 The specificities of demographics in Sub-Saharan Africa

The persistence of a high regime of fertility in Sub-saharan Africa, despite the important progress made in education and health since the 1960s, opens a room for other factors that may impact the relationship between human capital and fertility. Murtin (2009) estimated that when average schooling grows from 0 to 10 years, then fertility should decrease by about 50 to 80%. But this did not happen in SSA. A particular feature of SSA's family structure may hamper the demographic transition. Indeed in Sub-Saharan Africa, polygyny remains common and frequent. Polygyny is a particular form of marriage that has specific effects on fertility choice and education investment. The economics of polygyny was pioneered by Gary Becker (1974) and (1981) and Amyra Grossbard

[9] based on a panel of countries during 1870—2000

(1978). These papers focussed on the effects of polygamy on the marriage market, and, particularly, on the interactions with the different productivity level of women and men. Bergstrom (1994) expended these analyses. More recently, a stream of papers relating productivity with development (Jacoby, 1995) found that the development of polygyny is linked to the productivity of women, using micro data in agriculture from Ivory Coast. Tertilt (2005) argues that polygyny might be negatively related to development. The competition for wives in a polygamous society increases the bride price, which diverts savings from investment in physical capital and, thus, lowers the capital stock. Moreover, the incentives to have more children are high because the men receive the bride price on behalf of the daughters. In further analysis, Tertilt (2006) and Schoellman and Tertilt (2006) show in a calibrated model that changes in marriage law or women property rights regarding polygyny may have positive impacts on growth. These analyses show that polygyny plays definitively a role on the procreation behavior of families in Sub-saharan Africa and could delay the demographic transition. Sub-Saharan African countries are still experiencing high population growth rates. The population average growth rate in SSA was 2.5 percent in 2009. The average total fertility rate was 5.1 child per women in 2009[10]. Sub-Saharan African countries' fertility rates have decreased slightly since the early 1980's. Some expect that it could have declined more (see Figure 1 below).

Figure 1: Evolution of demographic patterns of Sub-Saharan Africa

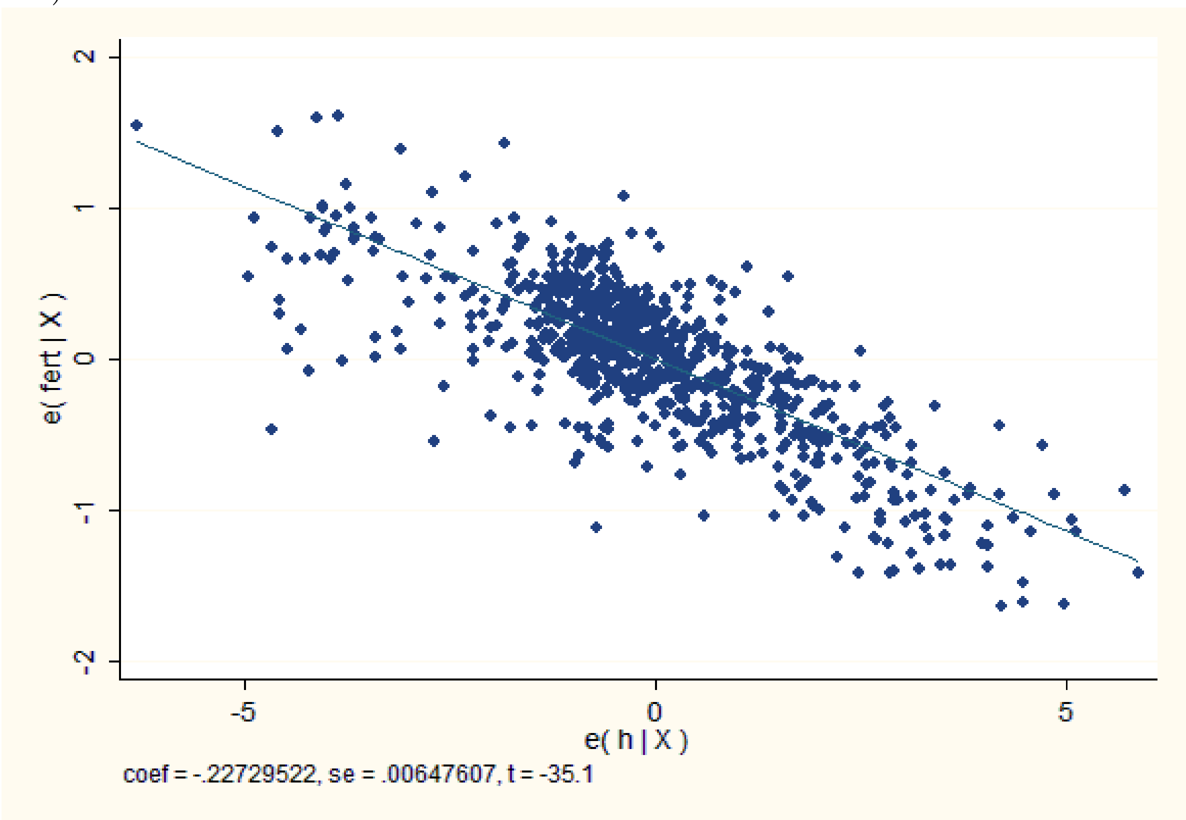


However, the progress made in the education and health sectors during the last twenty years have not been translated into lower fertility rates. Those changes may not be enough to trigger the demographic transition if the impact of the family structure is not taken

[10]Despite the effects of HIV/AIDS and its impacts on growth, see Cahu and Fall (2011).

into account[11]. Education decreases fertility through its negative impact on the marriages and its positive impact on contraception behavior and child health (see Figure 2). But, polygyny may increase the number of unions, and then offsetting the beneficial effects of education Also, family structure may also affect fertility and education behaviors directly [12].

Figure 2: The correlation between education and fertility (our sample of countries, regional level)



3 A theoretical model of polygamy and fertility nexus

Polygyny may have effects at the individual level and at the society level. Indeed, women in a polygamous family may compete in the number of children they have, seeking either social status or inheritance. Also, if polygyny increases the duration of union, it increases the exposure to the risk of pregnancy and the number of children. But this effect may be compensated by the fact that men with several wives have to split their time between their various partners. At the society level, polygyny could have substantial indirect effects. First, it may increase the number of marriages in the society as it becomes easier for women to find a suitable groom in a polygamous society. Second, it may also boost the customary number of children that both man and women desire to maintain their rank in

[11] See Kalemli-Ozcan (2010) for an analysis of the demographic trends of SSA countries.
[12] See Lambert and al. (2011) for a study of the impact of polygamy on family's education investment.

the society. As a consequence, in area where polygyny is frequent, monogamous couples may tend to have more children. The combination of these two elements could offset any eventual lower fertility of women living within a polygynous family. We build a theoretical model in a network framework that helps disentangling the different effects. Our model is a two-period one in which agents marry in the first period and produce offsprings in the second period. Consequently, we shall develop it in two parts. The first part will study the prediction of a marriage market and its implication for how polygyny affects nuptial rate, and the second part will analyze its implications for the effects of polygyny on individual and aggregate fertility.

3.1 A Hierarchical Mating Economy

Our setting consists of a non-empty finite set of individuals $N = \{i_1, \ldots, i_n\}$ divided into a set of men $M = \{m_1, \ldots, m_k\}$ and a set of women $W = \{w_1, \ldots, w_k\}$, each of equal size. Men and women are ranked according to some objective criterium (the ranking criterium may be wealth for men and education or beauty for women). Without loss of generality, we assume the rank of m_i to be higher than that of m_{i+1} and the rank of w_i to be higher than that of w_{i+1}, $i = 1, ..., k-1$. Each individual derives utility from having marital relationships with the opposite sex, and higher-ranked individuals are more desired as partners. A woman can have at most one partner, whereas a man can have multiple partners depending on whether polygyny is allowed or not. Each man has an optimal number of partners. We further assume that there is a social rank threshold below which a man cannot afford to get married. We let M_1 represent the set of men who are above this threshold and M_2 represent those below the threshold (M_2 may be empty). This setting defines what we call a hierarchical mating economy. This definition is more formally summarized below:

Definition 1 *A hierarchical mating economy is a list* $\mathcal{E}^{\succ} = (N = M_1 \cup M_2 \cup W, (s_j^*)_{1 \leq j \leq n}, \succ_m, \succ_w)$ *where:*

- s_j^* *represents the optimal number of partners for individual* i_j*;*
- $\succ_m$ *and* $\succ_w$ *are linear orderings on* M *and* W *representing the rankings of men and women, respectively.* $\succ_m$ *also represents women's preferences over men's ranks and* $\succ_w$ *represents men's preferences over women's ranks.*

As we mentioned previously, we shall assume that $s_j^* = 1$ if $i_j \in W$. Also, on the second interpretation of $\succ_m$and $\succ_w$, we remark that $\succ_m$ is not a ranking of the subsets of the set of men by women as it is often the case in traditional matching problems; $\succ_m$ is a ranking of individual (or singleton) men by women; similarly, $\succ_w$ is a ranking of individual women by men. For our purpose, we do not need a ranking of the subsets of the set of agents on each side of the market.

Our goal is to study the equilibrium matching of this economy under two alternative cultures, namely the monogamous culture where a man can have at most one partner, and the polygynous culture where a man may have multiple partners. Equilibrium is captured by the notion of pairwise stability as defined in Pongou (2009a). According to this notion, a marriage network or matching g, understood as a collection of links between men and women, is pairwise stable if: "(i) no individual has an incentive to sever an existing link she is involved in; and (ii) no pair of a man and a woman have an incentive to form a new link while at the same time possibly severing some of the existing links they are involved in." We provide a more formal definition of pairwise stability below.

Definition 2 *Let $(\succeq_i)_{i\in N}$ be a profile of preferences on the set of all possible matchings, and g a matching. We say that g is pairwise stable with respect to $(\succeq_i)_{i\in N}$ if:*
(i) $\forall i \in N$, $\forall (i,j) \in g$, $g \succ_i g \setminus \{(i,j)\}$.
(ii) $\forall (i,j) \in (M \times W) \setminus g$, if network g' is obtained from g by adding the link (i,j) and perhaps severing other links involving i or j, $g' \succ_i g \Longrightarrow g \succeq_j g'$ and $g' \succ_j g \Longrightarrow g \succeq_i g'$.

The following proposition says that there is a unique pairwise stable matching in this economy. It also provides a characterization of this matching in terms of the number of partners that each individual obtains.

Proposition 1 *There exists a unique pairwise stable matching in this economy. More precisely:*

- *Under a monogamous culture, each man m_i is matched with woman w_i if $i \leq k - |M_2|$, and all men m_i and women w_i such that $i > k - |M_2|$ are unmatched.*
- *Under a polygynous culture, m_1 is matched with the first $s_1 = \min(s_1^*, |W|)$ highest ranked women, m_2 is matched with the next $s_2 = \min(s_2^*, |W| - s_1)$ highest ranked women, and so on. Iterating, m_i is matched with the next $s_i = \min(s_i^*, |W| - \sum_{j=1}^{i-1} s_j)$ highest ranked women, $i = 2, ..., k - |M_2|$. And all men m_i such that $i > k - |M_2|$ and the remaining women are unmatched.*

Proof. The proof is constructive and follows the steps in Pongou (2009a). The unique pairwise stable matching is constructed as follows. Suppose that men and women are lined up according to their social rank. Under monogamy, the highest ranked man m_1 first proposes the highest ranked of his s_1^* most preferred women, who is w_1. The latter accepts since m_1 is her most preferred man. Afterwards, both leave the market. Now comes m_2's turn, who proposes to the highest ranked of his s_2^* most preferred women remaining in the market, who is w_2; the latter accepts, given that m_2 is her most preferred man remaining in the market, both leaving the market afterwards; and so on, until $m_{k-|M_2|}$ last matches with $w_{k-|M_2|}$ and leaves the market. By definition, all men m_i such that $i > k - |M_2|$ will not match, which automatically implies that all women w_i such that $i > k - |M_2|$ will not match either. One can easily prove that the described matching is the unique pairwise stable.

Under polygyny, m_1 first proposes each of his $s_1 = \min(s_1^*, |W|)$ most preferred women. The latter accept his proposal given that m_1 is their most preferred man. These newly matched individuals then leave the market. Afterwards, m_2 proposes each of his $s_2 = \min(s_2^*, |W| - s_1)$ most preferred women remaining in the market. The latter accept his proposal given that m_2 is their most preferred man remaining in the market, and these newly matched individuals leave the market afterwards. It follows by induction that man m_i $(i = 2, ..., k - |M_2|)$ matches with the next $s_i = \min(s_i^*, |W| - \sum_{j=1}^{i-1} s_j)$ highest ranked women remaining in the market. As under monogamy, it is easy to prove that the resulting matching is the unique pairwise stable. ■

We provide below an illustration of this result.

Example 1 *Consider the following hierachical mating economy with 5 men m_1, m_2, m_3, m_4 and m_5 and 5 women w_1, w_2, w_3, w_4 and w_5 where the demand for wives by men is $(s_1^*, s_2^*, s_3^*, s_4^*, s_5^*) = (2, 2, 1, 1, 1)$ and $M_1 = M$ (each man may marry). Under monogamy, the unique equilibrium matching, represented by Figure 3, is the one in which each man m_i matches with woman w_i. Under polygyny, in the unique equilibrium matching, represented by Figure 4, m_1 is matched with w_1 and w_2, m_2 is matched with w_3 and w_4, m_3 is matched with w_5, and m_4 and m_5 are unmatched.*

Now, suppose that the demand for wives by men is $(s_1^*, s_2^*, s_3^*, s_4^*, s_5^*) = (2, 2, 1, 1, 1)$ and $M_1 = \{m_1, m_2, m_3, m_4\}$ (m_5 cannot marry). Under monogamy, each man m_i will with woman w_i if $1 \leq i \leq 4$, and m_5 and w_5 will be unmatched (Figure 5). Under Polygyny, the unique equilibrium matching will still be the one represented by Figure 4.

We note that while the number of marriages is the same under monogamy and polygyny in the former economy, the situation is quite different in the latter economy, where there the number of marriages is greater under polygyny than under monogamy. We shall later generalize this result. We also note that in both economies, the monopolizing power of highest-ranked men deprives their lowest-ranked counterparts of wives.

Figure 3: Monogamy equilibrium

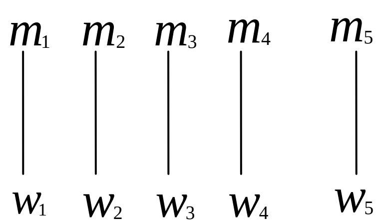


Figure 4: Polygyny equilibrium

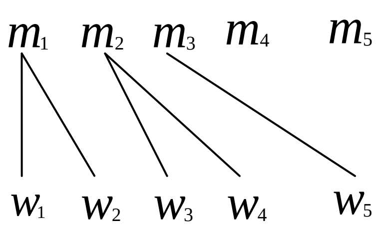
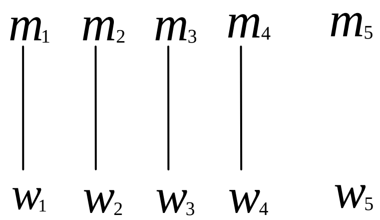


Figure 5: Monogamy equilibrium when M2 is non empty



A testable implication of Proposition 1 stated in Corollary 2 below is that the aggregate number of marriages (or nuptial rate) is higher under a polygynous culture than under a monogamous culture.

Corollary 2 *The aggregate number of marriages is higher under a polygynous culture than under a monogamous culture.*

Proof. Under a monogamous culture, the aggregate number of marriages equals the number of men who may get married, that is $|M_1|$. Under a polygynous culture, each man $m_i \in M_1$ may have at least one wife. So the aggregate demand for women by men who may get married is at least $|M_1|$. But it follows from the construction of the unique pairwise stable matching that arises in a polygynous culture in the proof of Proposition 1 that at least $|M_1|$ women get married, which implies that the number of marriages under polygyny is weakly greater than under monogamy. The inequality is strict if $s_i^* > 1$ for some man $m_i \in M_1$. ■

Another testable implication of Proposition 1 is that if the optimal number of partners that a man may have is increasing in his social rank, then higher-ranked women (or more beautiful women) have more chance to enter a polygynous relationship, with the number of co-wives increasing with social rank. This result is summarized in Corollary 3 below.

Corollary 3 *If $s_i^* \geq s_j^*$ whenever $i < j$, and if w_i and w_j are married, then the number of wives that w_i' s husband has weakly exceeds the number of wives that w_j' s husband has. The last inequality may be strict.*

Proof. The proof immediately follows from the construction of the pairwise stable matching in the proof of Proposition 1. ■

We note that a situation in which the number of women that a man may have increases with his social rank is when social rank is measured by wealth and wealth buys women (maybe in the form of bride price). Interestingly, Corollary 3 also implies that more beautiful women are more likely to be cheated upon by their husband. This is because more beautiful women marry wealthier men, who attract other women.

3.2 Beauty is Subjective

We consider a variant of a hierarchical mating economy in which men are ranked the same way by the women, but each man has his own ranking of women. The motivation here is that if the desirability of a woman as a partner is based, for instance, on how beautiful she is, each man may have a different appreciation of beautiful. For example, if beauty is determined by height, a man may not want his wife to be too much taller than him. Since each man generally has a different height, men will therefore rank women differently. We shall call a hierarchical mating economy in which individuals on one side have the same ranking of individuals on the opposite side, while the latter have different rankings of the former a hierarchical mating economy with one-sided subjective rankings.

Definition 3 *A hierarchical mating economy with one-sided subjective rankings is a list* $\mathcal{E}^{\succ} = (N = M_1 \cup M_2 \cup W, (s_j^*)_{1 \leq j \leq n}, \succ_m, (\succ_w^m)_{m \in M})$ *where:*

- s_j^* *represents the optimal number of partners for individual* i_j*;*
- $\succ_m$ *is a linear ordering on* M *representing the ranking of men by all women, and* $\succ_w^m$ *is a linear ordering on* W *representing the ranking of women by man* m*.* $\succ_m$ *also represents women's preferences over men's ranks and* $\succ_w^m$ *represents man* m*'s preferences over women's ranks.*

As for hierarchical mating economies, we find that a hierarchical mating economy with one-sided subjective rankings has a unique pairwise stable matching. We also give a description of this matching in terms of the number of partners that each individual obtains.

Proposition 4 *There exists a unique pairwise stable matching in a hierarchical mating economy with one-sided subjective rankings. More precisely:*

- *Under a monogamous culture,* m_1 *is matched with "his" highest ranked woman, each man* m_i *(*$i = 2, ..., k - |M_2|$*) is matched with "his" highest woman (not matched with* m_j*,* $j = 1, ..., i-1$*) if* $i \leq k - |M_2|$*, and all men* m_i *such that* $i > k - |M_2|$ *and the remaining women not matched with any man in* M_1 *are unmatched.*
- *Under a polygynous culture,* m_1 *is matched with "his"* $s_1 = \min(s_1^*, |W|)$ *highest ranked women,* m_2 *is matched with "his"* $s_2 = \min(s_2^*, |W| - s_1)$ *highest ranked women (not matched with* m_1*), and so on. Iterating,* m_i *is matched with "his"* $s_i = \min(s_i^*, |W| - \sum_{j=1}^{i-1} s_j)$ *highest ranked women (not matched with* m_j*,* $j = 1, ..., i-1$*),* $i = 2, ..., k - |M_2|$*. And all men* m_i *such that* $i > k - |M_2|$ *and the remaining women are unmatched.*

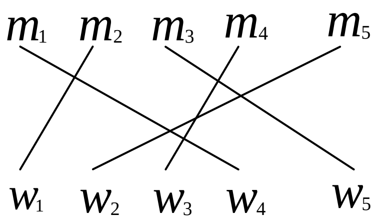
Proof. The reasoning is similar to that of Proposition 1 and so, the proof is left to the reader. ∎

We illustrate this result in the following example.

Example 2 *Consider the following hierachical mating economy, analyzed in Example 1, with 5 men* m_1*,* m_2*,* m_3*,* m_4 *and* m_5 *and 5 women* w_1*,* w_2*,* w_3*,* w_4 *and* w_5 *where the demand for wives by men is* $(s_1^*, s_2^*, s_3^*, s_4^*, s_5^*) = (2, 2, 1, 1, 1)$ *and* $M_1 = M$ *(each man may marry). The difference is that each man has his own ranking of women. Those rankings are the following:*

$m_1 : w_4 w_1 w_2 w_3 w_5$ *(that is,* m_1 *prefers* w_4 *over* w_1*,* w_1 *over* w_2*,* w_2 *over* w_3*, and* w_3 *over* w_5*)*

Figure 6: Monogamy equilibrium with one-sided subjective rankings

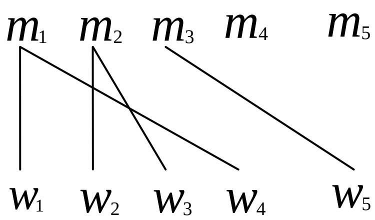


m_2 : $w_1w_3w_2w_4w_5$
m_3 : $w_1w_4w_5w_3w_2$
m_4 : $w_3w_2w_4w_1w_5$
m_5 : $w_2w_1w_3w_4w_5$

Under monogamy, the unique equilibrium matching, represented by Figure 6, is the one in which m_1 matches with w_4, m_2 matches with w_1, m_3 matches with w_5, m_4 matches with w_3, and m_5 matches with w_2. Under polygyny, the unique equilibrium matching, represented by Figure 7, is the one in which m_1 matches with w_4 and w_1, m_2 matches with w_3 and w_2 (his 2 highest ranked women not matched with m_1), m_3 matches with w_5, and m_4 and m_5 are unmatched.

We remark that the structure of the pairwise stable matching in terms of the distribution of links is the same for the first hierarchical mating economy analyzed in Example 1 and the hierarchical mating economy with one-sided subjective rankings being studied under either monogamy (Figure 3 has the same structure as Figure 6) or polygyny (Figure 4 and Figure 7 have the same structure), but the marriages are different.

Figure 7: Polygyny equilibrium with one-sided subjective rankings



As illustrated in Example 2, we note that the unique equilibrium matching arising in a hierarchical mating economy with one-sided subjective rankings has the same structure as the unique equilibrium matching arising in the corresponding economy hierarchical mating economy under either monogamy or polygyny. Indeed, both matching are similar up to permutations of the women, with men having the exact same number of women. This implies that the finding stated in Corollary 2, according to witch the number of marriages is greater under a polygynous culture than under a monogamous culture hierarchical mating economies, extends to hierarchical mating economies with one-sided subjective rankings as well. In the next section, we shall study the effect of polygyny on fertility at the individual and aggregate level.

3.3 The linkages between Polygyny and Fertility

In this section, we study the effect of polygyny on fertility at the individual level. We conduct this analysis under two alternative assumptions on the structure of preferences. First, we assume that children are the only consumption good in the household. Under the second assumption, parents derive utility not only from the number of children they have, but from other goods as well. Another salient feature of the model is that the number of children brings prestige to their parents, so that having more children while other parents have less generates more utility.

3.3.1 Children as the Only Good

Assume that a man m has l wives $w_1, ..., w_l$. Each individual derives utility from having children. A child is conceived out of the consent of his two parents, and is raised with resources contributed by both. For simplicity, we assume that they have identical preferences and endowment. Denote by u and y each individual's utility function and endowment (endowment includes all types of resources needed to raise a child such as financial resources, time, attention, etc.). We assume that u is twice-continuously differentiable and strictly concave and increasing in the number of children. Let c be the price of a child, n_m the total number of children born to the man m and all his wives, and n_i the number of children born to wife w_i $(i = 1, ..., l)$. It obviously follows that:

$$n_m = n_1 + ... + n_l \text{ and } cn_m = y + ly \tag{1}$$

Given that a child is conceived out of the consent of his two parents, it makes sense to assume that a man who has several wives decides how many children to give each wife. In fact, a wife cannot have more children than her husband wants to give her. Conversely, a husband cannot give any of his wives more children than the number she desires. But within our framework, we have assumed that man m and each of his wives have identical preferences, so that no wife desires more children than m. We shall therefore consider a unitary household model in which all incomes are pooled together and the husband, acting as a social planner, decides how many children (n_i) to give each wife w_i. It makes sense to assume that he allocates children across his wives so as to maximize a social welfare function such as the following:

$$U(n_m, n_1, ..., n_l) = u(n_m) + u(n_1) + ... + u(n_l) \quad [13] \tag{2}$$

[13] Note that our social welfare function is a bit different from traditional social welfare functions which do not incorporate the social planner's utility. In this respect, our social planner is not entirely "benevolent".

His maximization problem can be formulated as follows:

$$\begin{array}{rrcl} & MaximizeU(n_m, n_1, ..., n_l) & = & u(n_m) + u(n_1) + ... + u(n_l) \\ \text{subject to} & n_m & = & n_1 + ... + n_l, \\ & cn_m & = & y + ly, \\ & n_i & \geq & 0, i = 1, ..., l. \end{array} \tag{3}$$

It is easy to see that the solution of (3) is the egalitarian solution $n_i^* = n_w^* = \frac{y}{lc} + \frac{y}{c}$ for all $i = 1, ..., l$ and $n_m^* = \frac{y+ly}{c}$. Interestingly, we note that the functional form of n_w^* shows that each woman receives the number of children corresponding to her own endowment plus her husband's endowment shared equally across all wives. These results lead to the following testable implications, which say that the number of children that a man has increases with the number of wives he has, but the number of children that each wife has decreases with the number of wives.

Proposition 5 *n_m^* is strictly increasing in l and n_w^* is strictly decreasing in l.*

Proof. The proof comes from the expression of n_m^* and n_w^* above. ■

We note that it follows from Proposition 5 that a woman in a monogamous relationship has more children than a woman in a polygynous relationship. However, a man in a monogamous relationship has less children than a man in a polygynous relationship.

3.3.2 Envy or children as a signal of prestige

We now introduce envy or "others' regarding preferences" in the model. This may arise in a context in which the number of children is a source of prestige to their parents, so that having more children while other parents in the society have less generates more utility. More formally, if we let n_{i_j} be the number of children born to an individual $i_j \in N$, and n_{-m} the total number of children born to his/her neighbor, then the individual's utility is increasing in $n_{i_j} - \alpha n_{-m}$ where $\alpha > 0$ is the degree to which he/she envies his/her neighbor. In particular, if α tends to 0, there is little envy, a situation similar to our assumption in Section 3.3. We shall also assume that each individual derives utility from other consumption goods that we summarize into a single variable $x \in \mathbb{R}_+$. It follows that each individual's utility function is defined over the collection of bundles $(n_{i_j} - \alpha n_{-m}, x)$. For simplicity, we shall assume such a utility function to be additively separable, so that it can be written as:

$$u(n_{i_j} - \alpha n_{-m}) + v(x) \tag{4}$$

where each of the functions u and v is twice-continuously differentiable and strictly concave and increasing.

Following the same argument developed in the model without envy, we shall again consider a unitary household model where the husband, acting as a social planner, allocates children across his wives and the $x-$good across his wives and himself. If we let p be the price of the $x-$good, his maximization problem will now be:

$$\begin{array}{rrcl} & MaximizeU(n_m, n_1, ..., n_l, x_m, x_1, ..., x_l) & = & u(n_m - \alpha n_{-m}) + v(x_m) + u(n_1 - \alpha n_{-m}) \\ & & & +v(x_1) + ... + u(n_l - \alpha n_{-m}) + v(x_l) \\ \text{subject to} & n_m & = & n_1 + ... + n_l, \\ & cn_m + p(x_m + x_1 + ... + x_l) & = & y + ly, \\ & n_i & \geq & 0, i = 1, ..., l, \\ & x_m & \geq & 0, x_i \geq 0, i = 1, ..., l \end{array} \tag{5}$$

The following claims will be useful in the analysis of this maximization problem.

Claim 1: U attains a maximum in the constraint set.

Proof. It follows from the constraints that each $n_i \in [0, \frac{y+ly}{c}]$ and each $x_i \in [0, \frac{y+ly}{p}]$, $i = m, 1, 2, ..., l$. The constraint set therefore is $C = [0, \frac{y+ly}{c}]^{l+1} \times [0, \frac{y+ly}{p}]^{l+1}$, which is a closed and bounded subset of $\mathbb{R}^{2(l+1)}$. Hence, it follows from the Heine-Borel Theorem that C is compact. Given that U is a real-valued continuous function defined on a compact set, we conclude by the Bolzano-Weierstrass Theorem on the existence of extreme value that U attains a maximum in C. ■

Claim 2: Let f be a real-valued continuous function defined on a bounded interval $I \subset \mathbb{R}$. If f is strictly concave and increasing, then the function defined by $g(x_1, ..., x_n) = f(x_1) + ... + f(x_n)$ attains a unique maximum $(x_1^*, ..., x_n^*)$ in I^n $(n > 1)$. Furthermore, $x_1^* = x_2^* = ... = x_n^*$.

Proof. The proof is easy and left to the reader. ■

Given that U is additively separable, following Bergstrom (2011), (4) can be split up into the following maximizing problems:

$$\begin{aligned} &Maximize\ u(n_m - \alpha n_{-m}) \\ &\text{subject to} \quad cn_m = y_1 \end{aligned} \tag{6}$$

and

$$\begin{aligned} &Maximize\ u(n_1 - \alpha n_{-m}) + ... + u(n_l - \alpha n_{-m}) \\ &\text{subject to} \quad n_m = n_1 + ... + n_l, \\ &n_i \geq 0, i = 1, ..., l, \end{aligned} \tag{7}$$

and

$$\begin{aligned} &Maximize v(x_m) + v(x_1) + ... + v(x_l) \\ &\text{subject to} \quad p(x_m + x_1 + ... + x_l) = y_2, \\ &x_m \geq 0, x_i \geq 0, i = 1, ..., l, \end{aligned} \tag{8}$$

where $y_1 + y_2 = y + ly$. Here, income (y) is spent on children (y_1) and other consumption goods (y_2). But unlike in Section 3.3, the allocation of income between these two types of good is not fixed.

The solution of ($5'$) is $n_m^* = \frac{y_1}{c}$. It follows from Claim 2 that the solution of ($5''$) is $(n_1^*, ..., n_l^*)$ such that $n_1^* = n_2^* = ... = n_l^* = \frac{n_m^*}{l}$, and the solution of ($5'''$) is $(x_m^*, x_1^*, ..., x_l^*)$ such that $x_m^* = x_1^* = ... = x_l^*$

Since the egalitarian solution arises in equilibrium, pose $n_i = n_w$, $i = 1, ..., l$, $x_i = x$, and $i = m, 1, ..., l$. Our maximization problem then becomes:

$$\begin{aligned} Maximize\ U(n_w, x) &= u(ln_w - \alpha n_{-m}) + lu(n_w - \alpha n_{-m}) + (l+1)v(x) \\ \text{subject to} \quad cln_w + p(l+1)x &= y + ly \\ n_w &\geq 0 \\ x &\geq 0 \end{aligned} \tag{9}$$

Claim 1 and Claim 2 ensure that a unique equilibrium exists. We distinguish three cases: (a) $n_w = 0$; (b) $x = 0$; (c) $n_w > 0$ and $x > 0$.

If $n_w = 0$, then $x^* = \frac{y+ly}{p(l+1)}$. If $x = 0$, then $n_w^* = \frac{y+ly}{cl}$, which corresponds to the previously analyzed situation in which children were the only good.

If $n_w > 0$ and $x > 0$, then from the equality constraint, we deduce $x = \frac{y+ly-cln_w}{p(l+1)}$, which implies that the social planner's problem will simply consist of maximizing:

$$U(n_w) = u(ln_w - \alpha n_{-m}) + lu(n_w - \alpha n_{-m}) + (l+1)v(\frac{y + ly - cln_w}{p(l+1)}) \tag{10}$$

or equivalently

$$U(n_m) = u(n_m - \alpha n_{-m}) + lu(\frac{n_m}{l} - \alpha n_{-m}) + (l+1)v(\frac{y + ly - cn_m}{p(l+1)}) \tag{11}$$

Both functions will be useful for the comparative statics analysis. The first order conditions for these two functions are respectively:

$$U'(n_w) = lu'(ln_w - \alpha n_{-m}) + lu'(n_w - \alpha n_{-m}) - \frac{cl}{p}v'(\frac{y + ly - cln_w}{p(l+1)}) = 0 \tag{12}$$

and

$$U'(n_m) = u'(n_m - \alpha n_{-m}) + u'(\frac{n_m}{l} - \alpha n_{-m}) - \frac{c}{p}v'(\frac{y + ly - cn_m}{p(l+1)}) = 0 \tag{13}$$

We derive testable implications. First, the number of children that a man has increases with the number of wives he has, but the number of children that each wife may have increases or decreases with the number of wives depending on the utility function.

Proposition 6 *n_m^* is strictly increasing in l. n_w^* may be strictly increasing or decreasing in l depending on the utility function.*

Proof. 1) We want to show that at $n_m = n_m^*$, $\frac{\partial n_m}{\partial l} > 0$. First compute $\frac{\partial n_m}{\partial l}$ by applying the Implicit Function Theorem to (10). Write:

$U'(n_m) = u'(n_m - \alpha n_{-m}) + u'(\frac{n_m}{l} - \alpha n_{-m}) - \frac{c}{p}v'(\frac{y+ly-cn_m}{p(l+1)}) = f(n_m, l, n_{-m})$.

We have $\frac{\partial n_m}{\partial l} = -\frac{\partial f(n_m,l,n_{-m})}{\partial l} \times (\frac{\partial f(n_m,l,n_{-m})}{\partial n_m})^{-1}$. The reader can check that:

$-\frac{\partial f(n_m,l,n_{-m})}{\partial l} = -(-\frac{n_m}{l^2}u''(\frac{n_m}{l} - \alpha n_{-m}) - c^2 n_m v''(\frac{(l+1)y-cn_m}{p(l+1)}))$,

which is clearly < 0 given that $u'' < 0$ and $v'' < 0$ by assumption.

Also, we have:

$\frac{\partial f(n_m,l,n_{-m})}{\partial n_m} = u''(n_m - \alpha n_{-m}) + \frac{1}{l}u''(\frac{n_m}{l} - \alpha n_{-m}) + \frac{c^2}{p^2(l+1)}v''(\frac{(l+1)y-cn_m}{p(l+1)}) < 0$,

which implies that $(\frac{\partial f(n_m,l,n_{-m})}{\partial n_m})^{-1} < 0$. Since $\frac{\partial n_m}{\partial l}$ is the product of two negative numbers, it is positive.

2) Let us prove that at $n_w = n_w^*$, the sign of $\frac{\partial n_w}{\partial l}$ is ambiguous. Compute $\frac{\partial n_w}{\partial l}$ by applying the Implicit Function Theorem to (9). Write:

$U'(n_w) = lu'(ln_w - \alpha n_{-m}) + lu'(n_w - \alpha n_{-m}) - \frac{cl}{p}v'(\frac{y+ly-cln_w}{p(l+1)}) = g(n_w, l, n_{-m})$.

We have: $\frac{\partial n_w}{\partial l} = -\frac{\partial g(n_w,l,n_{-m})}{\partial l} \times (\frac{\partial g(n_w,l,n_{-m})}{\partial n_w})^{-1}$. The reader can check that:

$-\frac{\partial g(n_w,l,n_{-m})}{\partial l} = -(ln_w u''(ln_w - \alpha n_{-m}) + u'(ln_w - \alpha n_{-m}) + u'(n_w - \alpha n_{-m}) - \frac{c}{p}v'(\frac{(l+1)y-cln_w}{p(l+1)}) - \frac{c^2 ln_w}{p^2(l+1)^2}v''(\frac{(l+1)y-cln_w}{p(l+1)}))$.

Given that $u' > 0$ and $u'' < 0$, the sign of $-\frac{\partial g(n_w,l,n_{-m})}{\partial l}$ is clearly ambiguous. It could be positive or negative. However,

$\frac{\partial g(n_w,l,n_{-m})}{\partial n_w} = (l^2 u''(ln_w - \alpha n_{-m}) + lu''(n_w - \alpha n_{-m}) + \frac{cl}{p^2(l+1)}v''(\frac{(l+1)y-cln_w}{p(l+1)}) < 0$.

So if $-\frac{\partial g(n_w,l,n_{-m})}{\partial l} > 0$, then $\frac{\partial n_w}{\partial l} < 0$, and if $-\frac{\partial g(n_w,l,n_{-m})}{\partial l} < 0$, then $\frac{\partial n_w}{\partial l} > 0$. ■

We also analyze the impact of an individual's neighbor fertility on his fertility. We find that an individual's number of children is positively affected by the number of children his neighbor has, which is a contagion effect of fertility.

Proposition 7 *n_m^* is strictly increasing in n_{-m}.*

Proof. We study the sign of $\frac{\partial n_m}{\partial n_{-m}}$ at $n_w = n_w^*$. We apply the Implicit Function Theorem to (10). Write:

$U'(n_m) = u'(n_m - \alpha n_{-m}) + u'(\frac{n_m}{l} - \alpha n_{-m}) - \frac{c}{p} v'(\frac{y+ly-cn_m}{p(l+1)}) = f(n_m, l, n_{-m})$.

We have $\frac{\partial n_m}{\partial n_{-m}} = -\frac{\partial f(n_m,l,n_{-m})}{\partial n_{-m}} \times (\frac{\partial f(n_m,l,n_{-m})}{\partial n_m})^{-1}$. The reader can check that:

$-\frac{\partial f(n_m,l,n_{-m})}{\partial n_{-m}} = -\alpha[(-u''(n_m - \alpha n_{-m}) - u''(\frac{n_m}{l} - \alpha n_{-m})] < 0$.

Also, we have:

$\frac{\partial f(n_m,l,n_{-m})}{\partial n_m} = u''(n_m - \alpha n_{-m}) + \frac{1}{l} u''(\frac{n_m}{l} - \alpha n_{-m}) + \frac{c^2}{p^2(l+1)} v''(\frac{(l+1)y-cn_m}{p(l+1)}) < 0$,

which implies that $(\frac{\partial f(n_m,l,n_{-m})}{\partial n_m})^{-1} < 0$. We conclude that $\frac{\partial n_m}{\partial n_{-m}} > 0$. ■

Not surprisingly, we note from the expression of $-\frac{\partial f(n_m,l,n_{-m})}{\partial n_{-m}}$ in the proof that as the degree of envy (α) tends to 0, the marginal effect of an individual's neighbor fertility on his own fertility tends to 0 as well. So fertility is only as contagious as much as envy is strong.

As a corollary of Proposition 7, a monogamous individual in a functioning polygynous culture has more children than a monogamous individual in a monogamous culture.

Corollary 8 *A monogamous individual in a functioning polygynous culture has more children than a monogamous individual in a monogamous culture.*

Proof. The proof follows from the fact that in a functioning polygynous culture, there is at least one man who has several wives, and who by Proposition 6 has more children than he would have had in a monogamous culture. A monogamous individual in a polygynous culture is therefore exposed to the fertility behavior of such a polygynist, which by Proposition 7 has a positive effect on his own fertility. ■

3.4 Aggregate Number of Children in a Polygynous versus a Monogamous Culture

In this section, we investigate the effect of matrimonial culture on the total number of children in a society. Our analysis draws on the findings of the previous sections. We will distinguish two situations, namely one in which the number of children that a woman has decreases with the number of wives her husband has, and one in which the opposite holds. From the analysis conducted in previous sections, we know that the first situation occurs when children are the only consumption good, or under certain conditions, when children bring prestige to their parents. We will see that in such a situation, the effect of a polygynous culture on the aggregate number of children is ambiguous.

Let $(s_i^*)_{i\in M}$ be the demand for women by the men in a hierarchical mating economy. We know that $M = M_1 \cup M_2$ and $s_i = 0$ if $i \in M_2$. We have the following result.

Proposition 9 *Assume that* $\frac{\partial n_w^*}{\partial l} < 0$.

1) If $\sum\limits_{i \in M_1} s_i^* < |W|$*, then the total number of children is greater in a polygynous culture than in a monogamous culture. The inequality is strict if* $s_i^* > 1$ *for some man* $m_i \in M_1$.

2) If $\sum\limits_{i \in M_1} s_i^* \geq |W|$*, then the total number of children may be lower in a polygynous culture than in a monogamous culture.*

Proof. 1) If $\sum\limits_{i \in M_1} s_i^* < |W|$, meaning that the aggregate demand for women by the men who may get married is smaller than the total number of women, then obviously, each man in M_1 will obtain his optimal number of women in the unique equilibrium equilibrium that exists in the economy. Since each man in M_1 has at least one woman, by Propositions 4 and 5, each such man will have at least the number of children he would have had in a monogamous culture, implying that the total number of children is greater in a polygynous than in a monogamous culture. Assume that $s_i^* > 1$ for some man $m_i \in M_1$. By Propositions 4 and 5, given that m_i has more than one wife, he will have strictly more children than he would have had in a monogamous culture, which implies strict inequality when we compare the aggregate number of children under the two regimes.

2) If $\sum\limits_{i \in M_1} s_i^* \geq |W|$, all women will get married in a polygynous culture, but some men in M_1 may remain unmatched. Let us show by a simple example that the total number of children may be lower in a polygynous than in a monogamous culture. Consider a hierarchical mating economy that has 4 men m_1, m_2, m_3, and m_4 and 4 women w_1, w_2, w_3, and w_4, where $(s_1^*, s_2^*, s_3^*, s_4^*) = (2, 2, 1, 1)$ and $M_1 = M$ (each man may marry). Suppose that preferences over the number of children are such that a monogamous man gets 3 children and a man who has two wives gets 4 children, with each wife having 2 children (note that this assumption is consistent with $\frac{\partial n_m^*}{\partial l} < 0$). Under monogamy, the unique equilibrium matching is the one in which each man m_i matches with woman w_i. In this case, each couple has 3 children, and thus the total number of children is 12. Under polygyny, the unique equilibrium matching is one in which m_1 is matched with w_1 and w_2, m_2 is matched with w_3 and w_4, and m_3 and m_4 are unmatched. In this case, m_1 and m_2 will have 4 children each, and m_3 and m_4 will have no child, yielding a total of 8 children. We conclude that in this particular example, the total number of children is smaller under polygyny than under monogamy, despite the fact that a polygynist has strictly more children than a monogamist at the individual level. Note, however, that if $(s_1^*, s_2^*, s_3^*, s_4^*)$ were $(2, 1, 1, 0)$, the other assumptions remaining unchanged, the total number of children would have been 10 under polygyny and 9 under monogamy, and the conclusion therefore would have different. ■

In the second situation where the number of children that a woman has increases with the number of wives her husband has, we find that the total number of children is greater in a polygynous culture than in a monogamous culture.

Proposition 10 *Assume that* $\frac{\partial n_w^*}{\partial l} > 0$*. Then, the total number of children is greater in a polygynous than in a monogamous culture. The inequality is strict if* $s_i^* > 1$ *for some man* $m_i \in M_1$.

Proof. By Corollary21, we know that the number of women who get married is greater in a polygynous than in a monogamous culture. Since $\frac{\partial n_w^*}{\partial l} > 0$ by assumption, under polygyny, each such woman has at least the number of children she would have got under monogamy, which implies that the total number of children is greater in a polygynous than in a monogamous culture. If one such woman shares her husband with at least one woman (that is, $s_i^* > 1$ for some man $m_i \in M_1$), by the assumption that $\frac{\partial n_w^*}{\partial l} > 0$, she will have strictly more children than she would have had in a monogamous culture, which yields the strict inequality. ■

4 Empirical analysis of the link between polygyny and fertility

4.1 Descriptive statistics and construction of infant mortality

The data are taken from the DHS surveys. For each country, at least three surveys collected in different years are appended. Although polygyny seem to exist in every of the 48 countries of Sub-Saharan Africa, we have limited our study to a dozen of them. Marriage and fertility are supposed to vary both with time and age. To disentangle those two dimensions, we need to append at least three waves of surveys for each country. This limits our choices of countries. Some countries have enacted laws to ban polygyny, such exogenous variations of polygyny, could be useful in the analysis. However, it appears that legal changes rather follow changes in the customs rather than the contrary. The difficulties of several countries to pass ban laws (such as Uganda) or to enforce them (as in Senegal[14]) illustrates that reality. Only five countries have modified their law about polygyny, see table A in appendix. We tried to introduce in the sample comparable countries where polygyny has remained legal or unlawful. Madagascar happens to be the only country where polygyny is totally illegal and where three waves of the DHS surveys are available. Table 1 below presents the average statistics for some variables in the different countries. Polygyny appears to be very frequent. It is more widespread in Western Africa than in the rest of the continent. 42% and 46% of the women in Senegal and Benin respectively, are married to polygamous men while only less than 20% of the women in Eastern African countries do. Mortality stands for the number of dead children before age one over 1000 births. It is still very high, reaching 73.8 in Malawi and 69.3 in Tanzania. Most of the women are involved in a "couple relationship", the percentage of women in a couple range from 49% in Rwanda to 83% in Tanzania. If we add the column of women formerly in a couple, widows or divorced, it appears that being single is a relatively rare situation. Women are married young, before 20 year old in general in all countries. These different elements point the characteristics of the marital structure in the different countries. Infant mortality rate is supposed to increase gross fertility because parents make more children in order to insure themselves against the loss of a baby. As infant mortality is likely to depend also on individual factors, it could be worthy to retrieve some information about it from the surveys. An obvious choice is to use the *actual* rate of death among the children of the women whose fertility we try to study. However, such a variable is strongly endogenous. Because the number of children born is a discrete and small variable, the *actual* mortality rate of a woman is a very uncertain measure[15] of the *theoretical* probability of death of her young children. To use that information however,

[14]In Senegal, as in many other countries, man are suppose to choose a “polygamous” or “monogamous” status when they marry for the first time. However, it is common for men having chosen to be monogamous during their youth to marry latter a second wife. This law is difficult to enforce as the first spouse may have to choose between polygamy and a divorce.

[15]Especially for the women with a small number of children or no children at all

we will use a Bayesian method to build individual mortality rate (see appendix B for the details of the construction). The main assumption is that the probability of death of a child does not depend on her rank into the brotherhood. Infant mortality is therefore assumed to be independent of the number of children a woman has[16].

Table 1: Average statistics for different variables and countries

	polygyny	Fertility	Mortality	Involved	Formerly	Marital age	# obs.
Madagascar	0.03	2.79	42.0	0.67	0.13	18.5	38,644
Rwanda	0.11	2.71	53.0	0.49	0.16	20.0	28,293
Zimbabwe	0.15	2.42	32.8	0.61	0.13	18.7	20,942
Malawi	0.17	3.23	73.8	0.73	0.12	17.4	41,465
Ghana	0.24	2.82	38.6	0.70	0.08	18.8	20,012
Tanzania	0.26	4.01	69.3	0.83	0.10	17.6	60,556
Uganda	0.29	3.49	59.7	0.69	0.13	17.4	22,847
Cameroon	0.29	3.08	48.2	0.74	0.08	17.5	20,028
Ivory Coast	0.33	3.31	47.5	0.71	0.07	17.8	20,825
Nigeria	0.33	3.24	60.5	0.74	0.05	17.2	33,831
Senegal	0.42	3.59	47.5	0.80	0.05	17.1	29,505
Benin	0.46	3.18	49.8	0.74	0.05	18.2	29,504

"polygyny" is measured as the percentage of women whose husband has several spouses. "Mortality" stands for the number of children (over 1,000) who died before reaching the age of one. "Involved" is the percentage of women in a couple. "Formerly" is the percentage of women who are widows, divorced or separated. "Marital age" is the average age at first marriage. All statistics are computed for women between 15 and 49.

4.2 Polygyny and fertility at the micro level : a first estimation

Microeconomics regressions may allow to determine whether competition among spouses of a polygamous man increases fertility. We estimate the effects of polygyny on the total children born, whether they are still alive or not, i.e. the fertility rate at the individual level. This estimation is run for currently married woman only and for each country separately. The relevance of such an estimation is jeopardized by the fact that women entering a marriage with a polygamous man may have special characteristics. To tackle that issue, we introduce several controls such as the length of marriage, the number of times the woman was married, if she is non fecund and dummies variables indicating her religion and the area she lives in[17] as well. We also use the projected infant mortality rate at the individual level. We use OLS regressions (14), although the number of children is a discrete variable[18]

The fertility rate of a married woman i with characteristics X in a country j, denoted $\nu_m^{i,j}$, π is the boolean variable indicating whether the woman i lives with a polygamous man and t a linear yearly trend:

$$\nu_m^{i,j} = \gamma^j \pi^i + X^i \beta^j + \alpha^j t + \varepsilon^i \tag{14}$$

Estimations results are reported in table 2 :

[16]This assumption is probably a crude approximation as the first pregnancies in a woman's life, especially if they went wrong, are likely to induce complications during following ones.

[17]Using dummy variables for the region and the urban/rural location.

[18]Regressions were also run using ordered logit models, which gave similar results.

Table 2: Effect of polygyny on fertility at the individual level.						
Country	Benin	Ivory Coast	Cameroon	Ghana	Madagascar	Malawi
polygyny	-0.09^{**} (4.2)	0.15^{**} (3.8)	-0.18^{**} (4.5)	-0.12^{**} (3.6)	-0.09^{*} (3.3)	-0.14^{**} (4.9)
Marriage length	0.90^{**} (57.1)	0.78^{**} (41.1)	0.82^{**} (33.1)	0.76^{**} (41.8)	0.79^{**} (51.7)	0.83^{**} (46.9)
mort^r	28.70^{**} (25.7)	4.66^{**} (3.7)	7.50^{**} (5.2)	13.61^{**} (11.2)	12.50^{**} (22.4)	5.41^{**} (7.1)
mort^i	5.50^{**} (17.8)	4.24^{**} (8.8)	-0.81 (1.9)	7.54^{**} (15.2)	-0.58^{*} (2.8)	12.08^{**} (42.8)
# obs.	21590	12046	12962	12718	26065	21282
adj. R^2	0.67	0.55	0.53	0.62	0.66	0.57
Country	Nigeria	Rwanda	Senegal	Tanzania	Uganda	Zimbabwe
polygyny	-0.22^{**} (4.4)	$-0.08^{\dagger}$ (2.5)	-0.21^{**} (5.4)	-0.19^{**} (5.7)	0.06 (1.6)	-0.36^{**} (5.1)
Marriage length	1.25^{**} (52.1)	0.92^{**} (42.6)	0.81^{**} (39.4)	0.89^{**} (42.2)	0.74^{**} (40.1)	0.99^{**} (59.2)
mort^r	2.97^{*} (3.4)	16.39^{**} (16.6)	3.70^{**} (3.9)	6.16^{**} (4.7)	7.12^{**} (5.8)	17.80^{**} (15.9)
mort^i	5.90^{**} (15.5)	0.68 (1.5)	$0.79^{\dagger}$ (2.4)	0.64 (1.6)	11.21^{**} (17.3)	-1.71^{**} (4.3)
# obs.	9070	14441	13738	14865	12402	24282
adj. R^2	0.72	0.63	0.63	0.65	0.66	0.53

Dependent variable is the total number of children ever born. OLS regressions.
T-stats are between brackets. $^{\dagger}, ^{*}, ^{**}$ indicate significance at the 5%, 1% and 0.1% level.
Additional controls: Age, age^2, years of schooling, Rural, year trend, number of unions, "declared non fecond" dummy, religion and region dummies.

It appears that apart in Ivory Coast and Uganda, the practice of polygyny has a significant negative impact on fertility, once the duration of marriage is taken into account. It is to be noted that the number of spouses of the polygamous husbands has no significant impact on fertility. The duration of the union always increase fertility. Average infant mortality in the region ($mort^r$)[19] increases fertility in all countries as well. The Bayesian estimate of the probability of death of the children at the individual level ($mort^i$) is often positively correlated with fertility.

Several mechanisms could explain that all other things equal, women married with a man who has several spouses tends to have slightly less children. As pointed out by Pison (1986)[?] polygamous men may choose to marry less fecund (because of health condition or advanced age) women just to grant them a social status. A complementary explanation could be that, especially in urban areas, polygamous husbands are often not living with their spouses. Polygyny may also be seen as a way to compensate for longer postpartum or breast-feeding period, which slows down the rhythm of pregnancy.

4.3 Direct effects of polygyny on fertility at regional level

To exhibit external effects of the practice of polygyny on fertility, we make the following assumption. We assume that nuptial rate, the age of first marriage and eventually fertility depend on social characteristics as well as norms of the region the woman is living in. However, the typical size of a DHS household survey does not usually allow to split countries into more than 20 areas, where one can calculate representative average of demographic figures. To underline the external effects of polygyny, we have to pool data

[19] We calculate the average infant mortality in the region at the age of 20 for women who does not had children.

for different countries. We define 140 geographical areas r by splitting the 12 countries into regions. As we can compute the incidence of polygyny and other representative social characteristics in those areas at different point in time, we are able to obtain 559 different groups $k = \{r, t\}$ of women (among 366,000 individual observations) by pooling the women living in the same region r and interviewed during the year t. The average number of individual in each group is about 650. The "local" social and cultural characteristics y_t^r are calculated by averaging the individual characteristics (such as polygyny or religion) within the groups k.

$$y_t^r = y^k = \frac{1}{\#k} \sum_{i \in k} y^i \tag{15}$$

Seeking a direct link between fertility and polygyny, we investigate whether or not fertility is higher in areas where the incidence of polygyny is also higher. We therefore add $polygyny^k$, the average incidence of polygyny for a given year and in a given region. We control also by the average human capital $education^k$ and the shares of the population affiliated to the main religions b in each group k, R_b^k. Descriptive statistics of the social/cultural characteristic by group are given in table 3:

Table 3: Descriptive statistics of social/cultural characteristics at the local level

Variable	# Obs.	Mean	Std. Dev.	Min	Max
$polygyny^k$	547	0.25	0.14	0.00	0.63
$education^k$	559	4.69	2.15	0.26	10.24
$Catholic^k$	504	0.25	0.19	0.00	0.85
$Protestant^k$	504	0.21	0.19	0.00	0.93
$Muslim^r$	504	0.29	0.33	0.00	1.00

Religions and schooling characteristics at the "local" level are embedded in the vector Y^k. The total fertility of a women i from the group k is given by:

$$\nu_m^{i,k} = \gamma \pi^i + X^i \beta + \zeta polygyny^k + Y^k \theta + \alpha t + \varepsilon^i \tag{16}$$

The effects of social norms/culture are measured by the vector θ. The "external" effect of polygyny on fertility is given by the coefficient ζ. We also use OLS regressions. Results are reported in table 4:

Table 4 : Direct external effects of polygyny on fertility								
Dep. var.	——— total number of children ever born (OLS) ———							
$polygyny^k$	1.03^{**} (21.8)	0.81^{**} (18.5)	1.09^{**} (24.1)	0.59^{**} (7.4)	0.89^{**} (17.5)	0.71^{**} (15.0)	1.01^{**} (20.7)	0.58^{**} (6.4)
$education^k$	0.05^{**} (14.7)	0.02^{**} (6.0)	$0.01^{\dagger}$ (2.4)	0.02^{**} (4.0)	0.05^{**} (13.3)	0.02^{**} (4.7)	$0.01^{\dagger}$ (2.3)	0.01^{*} (2.8)
$mort^r$	10.81^{**} (44.6)	5.69^{**} (24.9)	5.22^{**} (22.8)	6.31^{**} (23.7)	10.58^{**} (39.2)	5.40^{**} (21.4)	5.01^{**} (19.9)	5.59^{**} (19.0)
$mort^i$	4.48^{**} (34.0)	3.64^{**} (29.7)	3.58^{**} (29.2)	3.55^{**} (28.7)	4.13^{**} (26.4)	3.34^{**} (23.1)	3.29^{**} (22.8)	3.20^{**} (22.0)
Marriage length	-	yes	yes	yes	-	yes	yes	yes
$Religion^k$	-	-	yes	yes	-	-	yes	yes
Country	-	-	-	yes	-	-	-	yes
# obs.	126167	126167	126167	126167	93946	93946	93946	93946
adj. R^2	0.57	0.63	0.63	0.64	0.59	0.65	0.65	0.65
Sample	All women in a relationship				Unique spouse only			

T-stats are between brackets. †,*,** indicate significance at the 5%, 1% and 0.1% level.
Additional controls: Age, age^2, years of schooling, Rural, year trend, Body Mass Index, "declared unfecund" dummy, religion dummies.

The direct effect of polygyny on fecundity seems to be strong, whatever the configuration used. Infant mortality, both at the local and individual level appears to be highly correlated with fertility. The social (external) effect of polygyny decreases when controlling by the duration of the union. The practice of polygyny may lengthen the unions, presumably by lowering the age at which girls get married. Although polygyny is correlated with religious beliefs, it appears that polygyny on itself affects fertility, within both polygamous and monogamous households. As the social environment of the area is described by the average education and infant mortality, it is likely that the practice of polygyny witnesses rather local cultural particularities.

4.4 Effects of polygyny on nuptial rate, remarriage and age of first marriage

Polygyny could increase the share of married women. To test this assumption, we model the probability of being married with a probit model, equation (17). We assume that the probability of marriage depends on individual characteristics X^i but also on the local context Y^k and the incidence of polygyny $polygyny^k$. We control by age and education but also by the body mass index and if the women has been declared non fecund, as more healthy women may marry more easily. We control also by the average level of education among women, which captures the fact that women are more free to refuse (early) marriage in a society where they are collectively empowered by the education[20]. Religions dummies are also embedded in the vector Y^k.

$$p_n = \Phi\big(X^i\beta_n + \zeta_n polygyny^k + Y^k\theta_n\big) \tag{17}$$

The probit model is estimated using the pooled data. Results are reported in the three left columns of table 5. It appears that whatever the controls used, women happen to be

[20] The causality between women education and marriages probably goes in both directions as societies more prone to gender equality and women independence probably also favor their education.

more frequently involved in a union in areas where polygyny is more frequent. Moreover the effect of polygyny is very significant.

Polygyny can also affects the share of women staying single after the end of an union, p_f because of the death of their husband, a divorce or a separation. We also use a probit model to estimate the effects of polygyny on the probability of staying single after having been formerly married (three right columns of table 5).

$$p_f = \Phi\big(X^i\beta_f + \zeta_f polygyny^k + Y^k\theta_f\big) \tag{18}$$

It appears that again the incidence of polygyny increases the probability of remarriage. This variable always remain very significant. Women do remarry very frequently in polygamous societies. A patriarchal culture could explain at the same time polygyny and the importance of being married for a woman. However, remarriages are practically possible because there is more room, especially among middle age men[21] to accommodate those unions in a polygamous society. It also fasten the process.

Table 5: Effects of polygyny on nuptial

Dep.	Prob. of not being single			Prob. of being divorced/widow		
$polygyny^k$	0.73^{**}	0.80^{**}	1.01^{**}	-1.45^{**}	-1.44^{**}	-0.99^{**}
	(23.6)	(24.6)	(16.6)	(36.5)	(35.1)	(12.8)
$education^k$	-0.02^{**}	-0.03^{**}	-0.04^{**}	0.00	0.00	0.00
	(8.1)	(11.3)	(13.0)	(0.4)	(0.6)	(1.0)
$Religion^k$	-	yes	yes	-	yes	yes
Country	-	-	yes	-	-	yes
# obs.	179420	179420	179420	144720	144720	144720
pseudo R^2	0.22	0.24	0.24	0.05	0.05	0.06
Sample	All women			Excluding never married women.		

Probit regressions.

T-stats are between brackets. †,*,** indicate significance at the 5%, 1% and 0.1% level.

Additional controls: Age, age^2, years of schooling, Rural, year trend, Body Mass Index, "declared non fecond" dummy, religion dummies.

Although earlier weddings do not necessarily mean that women will plan to have more children, this practice could increase population growth. First, as younger women tend to be more fecund, it may increases the number of pregnancies. And second, even if this does not increase the total net fertility [22], it is likely to reduce the time between two consecutive generations. Indeed for a given fertility rate, the demographic growth tend to accelerate when mothers are younger. The practice of polygyny induces an unbalances between men and women which push the girls to marry more quickly. To check that hypothesis, we regress, using OLS estimation, the age at which a women marries $a_m^{i,k}$ on individual and social characteristics, equation (19).

$$a_m^{i,k} = X^i\beta_m + \zeta_m polygyny^k + Y^k\theta_m + \varepsilon^i \tag{19}$$

The results are reported in table 6, first, for all the women between 15 and 49 and, second for women over 30 only. Overall, marriage happens indeed earlier in areas where polygyny is more frequent. However, the effect of polygyny is not always significant when measured on all women. But this sample is probably not relevant as it induces a selection

[21] As women tend to survive their husband, they are likely to remarry.

[22] That is the average number of surviving children per woman.

bias. Obviously, age at first marriage is only measured for women who already married. Therefore, when including the younger contestants, the places where women get married later are under-represented. To solve this problem, we estimate the same relation for women over 30 only[23]. On that sample, the correlation between polygyny and the age at which women marry is larger and much more significant.

There appears to be a strong correlation between the practice of polygyny and both marriages and fertility. In the following section, we will try to address the question of causality in two dimensions. First we will look at potential omitted variables which could cause at the same time polygyny and nuptial/fertility.

Table 6: Effects of polygyny on women's marriage age

Dep.	Age at first marriage (OLS)					
polygyny^k	-0.16 (1.9)	-0.34^{**} (3.9)	-1.19^{**} (7.5)	-1.20^{**} (8.0)	-1.33^{**} (8.5)	-1.39^{**} (4.9)
education^k	0.03^{**} (4.1)	0.07^{**} (10.7)	0.17^{**} (19.0)	-0.02 (1.9)	$0.02^{\dagger}$ (2.2)	0.17^{**} (10.4)
Religion^k	-	yes	yes	-	yes	yes
Country	-	-	yes	-	-	yes
# obs.	144720	144720	144720	63059	63059	63059
adj. R^2	0.1	0.1	0.14	0.09	0.09	0.12
Sample	Once married women			Once married women over 30		

T-stats are between brackets. †,*,** indicate significance at the 5%, 1% and 0.1% level. Additional controls: years of schooling, Rural, year trend, Body Mass Index, weight, religion dummies.

5 Identification strategy to instrument Polygyny: the height effect

5.1 Marriage and woman's height at the micro level

To deal with endogeneity we consider in this section the impact of women's height on both polygyny and fertility. Interestingly as reported in the DHS the height of women seem to matter for marriage. Two complementary explanations can be brought forward to explain this phenomenon. Height is known to be correlated with health status during childhood. As a consequence, tall women can be better off first because their height can be seen as a sign of good health. As beauty is also very likely to be correlated with health, taller women could also be regarded as prettier, leading to more frequent marriages. At the micro level, regressions support the claim that taller women are more valued as spouses. If this assumption is to be true, then a population with a larger proportion of tall women, all other things equal (especially on the man side), is likely to have a higher rate of marriage. The effect of height on polygyny is *a priori* ambiguous but one suggests that taller women can favor polygyny for two symmetric reasons:

(i) Let us assume that polygamous men have unobservable characteristics which make them more likely to marry. If polygamous men are choosier than monogamous ones and tall women are more valued, the pool in which polygamous men choose their brides from will be larger in a society with taller women. In a matching perspective, taller women should increase the incidence of polygyny.

[23] In average, women get married for the first time around 20.

(ii) If we assume conversely that polygamous men have unobservable characteristics which make them more valuable for brides, taller women, if more valued, are likely to be pickier about the choice of their spouse and will also prefer to marry a polygamous man.

To identify the causal link between polygyny and fertility we make the following assumptions:

Taller women are more valued in the perspective of an union

A higher share of tall women in the population favors both marriage and polygamy

To challenge the first assumption, we run regressions at the micro level about the probabilities of being married, being married with a polygamous man, the average age of first intercourse and marriage and the number of children. The results presented in table 7 support indeed the first assumption.

Table 7: Effects of height on marriage and fertility (Individual regressions)

	Probit In a couple	Probit In polygamous union	Probit Declared non fecund	OLS Age 1st intercourse	OLS Age 1st Marriage	OLS Fertility (# born child.)
Age	0.05*** (125.0)	0.02*** (45.6)	0.05*** (60.4)	0.32*** (199.8)	0.09*** (78.7)	0.26*** (428.8)
Year	−0.05*** (52.3)	−0.004*** (3.8)	0.03*** (15.4)	−0.17*** (43.1)	0.001 (0.5)	−0.07*** (57.5)
Schooling	−0.04*** (40.7)	−0.03*** (18.4)	−0.01*** (5.3)	0.08*** (23.6)	0.23*** (97.5)	−0.04*** (33.4)
Rural	0.29*** (33.1)	0.19*** (19.4)	−0.05** (3.3)	0.2*** (6.1)	−0.5*** (23.5)	0.35*** (31.6)
Muslim	0.32*** (28.1)	0.29*** (15.1)		0.2*** (4.5)	−0.72*** (25.4)	−0.01 (1)
Cathol.	−0.04*** (4.7)	−0.10*** (9.0)		0.03 (0.8)	0.19*** (7.9)	−0.02* (2)
Tradi.	0.23*** (10.0)	0.31*** (16.5)		−0.14 (1.8)	−0.42*** (8.9)	0.05* (2.3)
Height	2.39*** (14.6)	0.58** (3.1)	−2.41*** (7.3)	15.6*** (23.3)	3.02*** (6.9)	−0.06 (0.3)
# union>1						−0.37*** (30.4)
Ideal # child.						0.003*** (17)
Age 1st marr.						−0.21*** (157.4)
polygyny						−0.04** (3.3)
R2	0.16	0.11	0.14	0.21	0.17	0.64
# obs.	179944	125967	179834	176894	143632	125450

Controls: country fixed effects.

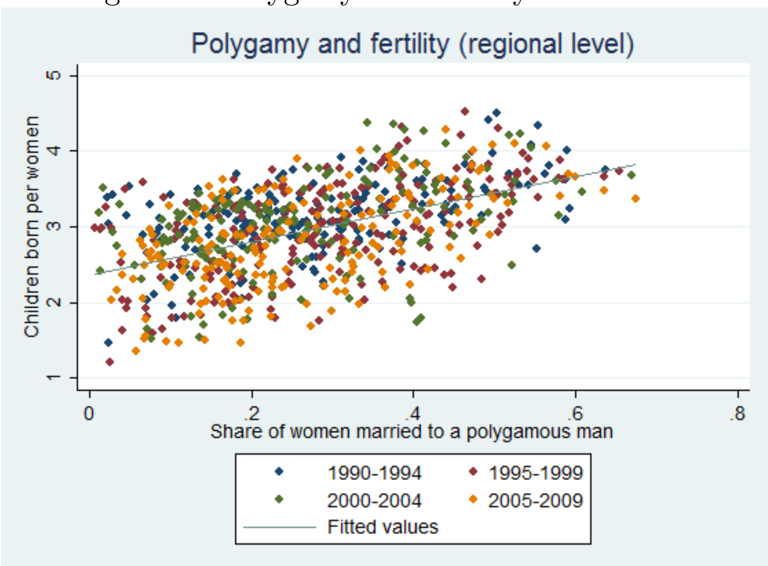
More specifically:

- Taller women get more often married, once taken into account other important factors such as education, age, religion and location.

- But taller women experience in average their first intercourse latter and also tend to marry when they are older. Taller women seem indeed choosier as they take more time to marry while having a higher probability of union.
- Taller women are more rarely declared non fecund which may provide a rationale for their value as spouses in societies which put emphasis on children.
- Taller women are also more married to polygamous husbands. Height seems to be valued by indeed by both polygamous and monogamous husbands.
- When controlling for polygyny, the height of women has no influence on the number of children at the individual level: tall women do not have more children all other things equal. As a consequence, height seems to be a good instrument of polygyny.

As height is correlated with health status during childhood, one could argue that the correlation between height and fertility just traduce the fact that healthier women are more likely to deliver a baby. However, first we control for infant mortality, which is strongly correlated with health quality and second height is not correlated with fertility once polygyny is taken into account.

Figure 8: Polygamy and fertility correlations



5.2 Polygyny and fertility causality at regional level instrumented by height

Using all available DHS surveys in Sub-saharan Africa, we pooled together 652 observations from 34 countries. For most of the countries, two or three waves of surveys are available. After controlling by age, education, religion, infant mortality, year of survey and age of marriage and country fixed effects, the incidence of polygyny happens to be positively correlated with fertility. In other words, in regions where polygyny is more frequent, the fertility rates tend to be higher (see table 8). To instrument polygyny, we regress the average share of married women within a polygamous household on age, education, main religions, year of survey and the average height of interviewed women and country fixed effects. Average height is very significantly correlated with the incidence of polygyny at the regional level. According to the instrumental regression, the impact of polygyny on fertility is much stronger that OLS regressions. The coefficient is also less significant because unfortunately the height is not a very strong instrument at regional level. In conclusion, it is likely that the impact of polygyny on fertility is indeed causal and its magnitude cannot be neglected.

Table 8: External effect of polygyny on fertility (Regional regressions)

	OLS polygyny	OLS Fertility	IV Fertility
polygyny	–	0.52^{***} 4.4	1.54^{*} 2.3
age	0.01 1.6	0.20^{***} 17.6	0.19^{***} 15.8
rural	0.01 0.6	0.42^{***} 9.3	0.40^{***} 8.4
Schooling	-0.03^{***} 10.9	-0.06^{***} 5.6	-0.03 1.4
height	0.0014^{***} 4.7	0.0014 1.6	–
year	0.0014 1.7	-0.01^{**} 2.6	-0.01^{**} 3.0
traditional	-0.07^{*} 2.4	-0.21^{*} 2.5	-0.14 1.3
muslim	0.01 0.4	-0.16^{**} 2.8	-0.17^{**} 2.8
catholic	-0.06^{*} 2.3	-0.20^{**} 2.8	-0.14 1.6
mortality	0.15 1.6	1.65^{***} 5.9	1.52^{***} 5.0
Age at first marriage	–	-0.12^{***} 11.0	-0.21^{***} 157.4
R2	0.74	0.88	0.87
# obs.	652	652	652

Controls: country fixed effects.

Regressions using variables averaged at the regional level by year of survey.

5.3 Robustness check

The correlation between fertility and polygyny is mostly due to the fact that women living in areas where the practice of polygyny is frequent tend to marry more and to have more babies. As women with a monogamous spouse also tend to have more children in those areas, it is very unlikely that higher fertility directly causes polygyny. We have estimated a direct causal effect of polygyny on fertility in the previous section, but that does not rule out the possibility that both fertility and polygyny could be induced by a third (unobservable) factor. In this section one address that hypothesis, first by proposing variables to capture what could be cultural traits favoring gender differentiation, fertility and the male domination and, second by estimating there impact on the fertility/polygyny relationship.

5.3.1 Conservatism: Men behaviors and polygyny

We make the underlying assumption that societies dominated by male interests tend to confine women in the reproductive function. In such societies, spouses and thus children are seen as exterior signs of wealth and male tend to accumulate both to compete in the society. If this assumption is valid, variables related to gender discrimination and male domination should explain both the levels of fertility and the practice of polygyny. As those variables need to capture social norms and customs, one wants to compute indicators averaged at the local level S_t^{jr}. By introducing such variables into the previous equation, we will be able to check the robustness of the estimates. If the incidence of polygyny π_t^r become insignificant while controlling by the set of variables $(S^j r_t)$As polygyny is correlated with lower age of marriage, higher fertility and lower schooling for women, it is tempting to search for indicators of some "conservative" sensitivity, in the sense that tasks and roles for men and women within the household and the society are much polarized. Fortunately, the DHS surveys provide a wealth of questions, to both men and women, allowing capturing cultural values and behaviors. We retained seven kinds of indicators to measure cultural beliefs and behaviors:

- **Age at which men marry**: If marriage is considered as an exterior sign of wealth, then men need to get richer to be able to marry and have children. In such society, the competition for brides is supposed to delay men's wedding as young people tend to be poorer. Because men marry latter there is a structural unbalance between the number of men and women able to marry which could support the practice of polygyny.

- **Men out of job**: In the same spirit, men without a job are less likely to find a bride and to sustain a big family.

- **Men with tertiary education:** A society which values education is less likely to put emphasis on the size of household as a sign of success. Societies with an educated elite may lead to different norms toward polygyny and fertility.

- **Faithfulness** (of husbands) may be considered as an indicator of machismo. In the DHS, the male contestants are indeed asked to report the number of sex partners (beside their spouses) they had during the last 12 months. We consider a man to be unfaithful if he had sex with a woman which is not his legitimate partner. In this case $u^i = 1$ and 0 otherwise.

- **Bias toward male babies**: Another way to catch a bias toward men in the society is to look at preferences regarding the gender of babies. In the DHS, contestants (both men and women) are asked about the "ideal" number of boys $\#_{boys}^*$and girls $\#_{girls}^*$they would like. This allows calculating a gender-bias indicator for men $mgb$and women wgbequation (20):

  $$\{mgb, wgb\} = \frac{\#_{boys}^* - \#_{girls}^*}{\#_{boys}^* + \#_{girls}^*} \tag{20}$$

  When there is no bias, xgb is equal to zero. It is positive for a bias toward boys and negative for a bias toward girls.

- **Occupation of men**: Labor intensive professions such as agriculture or trade may push men to marry several times to produce workforce for their business.

- **Tolerance to domestic violence**: In the DHS women were also asked if they found justified that a husband beat his wife if she refuses to have sex. The tolerance of domestic violence from the women side is another side of male domination.

- **Duration of breast-feeding**: Longer breast-feeding period is often associated with longer time of amenorrhoea[24] and abstinence. Longer duration of breast-feeding may in turn favor polygyny as well. The partial correlations (regional averages) of those variables are reported in table 9. Except for the duration of breast-feeding and the age of marriage for men, the polygyny and fertility variables tend to be correlated as expected with the "cultural" indicators.

Table 9: Partial correlations between being in couple and cultural variable

	In a couple	Formerly married	polygyny
Tolerance to beating	0.45	−0.16	0.48
Man male bias	0.13	−0.30	0.44
Breast-feeding duration	0.16	0.26	−0.06
Cheating	0.15	0.19	0.16
Man wedding age	−0.27	−0.14	0.15
Woman male bias	0.18	−0.36	0.46
Man schooling	−0.39	0.03	−0.34
Man tertiary education	−0.25	0.01	−0.13
Man without job	−0.38	−0.02	−0.27

5.3.2 Controlling for a "gender-biased" sensitivity

The above variables are therefore good candidates to measure some "gender-biased" or conservative sensitivity. To test whether the correlation within polygyny and fertility is due to a culture of conservatism one introduces the previous indicators (still averaged at the regional level) into the regressions of fertility or marriage at the individual level along with the average incidence of polygyny at the regional level as in regressions (16), (18) and 19) with S^k being a sub-vector of Y^k. The results are reported in table 10.

[24] Absence of menstrual period, often induced by a recent pregnancy and breastfeeding.

Table 10: Effects of gender-biased sensitivity on fertility and marriages.

Dep. Var	Fertility (# born child.)	Never * married	Formerly* married	Age 1st marriage
Unfaithful^k	0.4 ** (13.2)	-0.58^{**} (16.1)	-0.01 (0.5)	1.3 ** (11.9)
Men boys bias^k	1.65^{**} (9.9)	-0.49^{**} (3.0)	-1.26^{**} (7.8)	-5.36^{**} (9.0)
No work^k	0.3^{**} (5.1)	1.35^{**} (26.0)	0.28^{**} (5.5)	1.7^{**} (8.5)
Tertiary^k	-0.33^{**} (4.7)	0.1 (1.5)	-0.23^{**} (3.5)	0.58^{**} (2.3)
Women boys bias^k	-2.14^{**} (10.3)	3.16^{**} (16.7)	-2.03^{**} (10.2)	14.99^{**} (20.6)
Violen. Tolerance^k	0.54^{**} (9.3)	-0.19^{**} (2.9)	0.36^{**} (6.4)	-1.67^{**} (8.4)
Breast-feeding^k	-0.01^{**} (4.3)	-0.02^{**} (6.1)	0 (1.1)	0.17^{**} (14.6)
polygyny^k	0.96^{**} (11.8)	-0.19^{*} (2.4)	-0.99^{**} (12.9)	-1.69^{**} (6.0)

$\star$ indicates probit (vs OLS) regressions. Controls: age, age^2, schooling, religion (individual and local), average schooling at the local level and country fixed effects.

It appears that although all those variables explain at the same time polygyny and nuptial/fertility, they do not explain the essence of the correlation between polygyny and fertility. Although those variables may be only crude measures of the cultural traits of the population, it shows that the correlation between fertility and polygyny is unlikely to be a pure artifact.

6 Simulating the impact of polygyny at the macro level

At the micro level, polygyny reduces the gross fertility rate of married women. At the same time, a rise in the share of women living within a polygamous household both increases average fertility and the frequency of marriage. As women in a union have far more children that the single ones, polygyny may increases the overall fertility rate. The impact of polygyny on the average fertility rate is therefore the sum of three contradictory effects:(i) polygyny reduces fertility within the married couple but increases average fertility (ii) both directly and (iii) through more frequent union.

To calculate the net effect of polygyny, let us consider the women with the vector of characteristics X, whose marital status s can be never married (n), currently married or in a relationship (m) or single but formerly married (f). Let us designate ν_X^s the fertility rate of the woman with characteristics X and marital status s. Let us also denote p_X^s the probability that the marital status of such a woman be s. The average fertility rate of a woman with characteristics X, ν_X is therefore:

$$\nu_X = \nu_X^n p_X^n + \nu_X^f (1 - p_X^n) p_X^f + \nu_X^m (1 - p_X^n)(1 - p_X^f) \tag{21}$$

Let us denote π the local share of women married to a polygamous man. Polygyny is not to influence the fertility rate of unmarried woman directlyThat is $\frac{d\nu_X^n}{d\pi} = 0$. The marginal effect of the incidence of polygyny on fertility is therefore:

$$\begin{aligned}\frac{d\nu_X}{d\pi} &= (1-p_X^n)\left(\frac{d\nu_X^f}{d\pi}p_X^f + \frac{d\nu_X^m}{d\pi}(1-p_X^f)\right) + \frac{dp_X^n}{d\pi}\Big(\nu_X^n - \nu_X^m - p_X^f(\nu_X^f - \nu_X^m)\Big) \quad (22)\\ &+\frac{dp_X^f}{d\pi}(\nu_X^f - \nu_X^m)(1-p_X^n)\end{aligned}$$

The first term measures the direct effect of polygyny on fertility. The second term represents the effect of polygyny on fertility via the raise in the nuptial rate, whereas the third term stands for the effect of polygyny on the probability of remarriage. From this equation, one can derive the simulated equation which is the expected effect computed using the previous estimated probabilities and effects. It writes as follows (see appendix C for the determination):

$$\begin{aligned}E\left[\frac{d\nu}{d\pi}\right] &\approx (1-p^n)\frac{d\nu^m}{d\pi} - \frac{dp^n}{d\pi}\Big(E_X[\nu^m] - E_X[\nu^n] - p^f(E_X[\nu^m] - E_X[\nu^f])\Big) \quad (23)\\ &-\frac{dp^f}{d\pi}(1-p^n)(E_X[\nu^m] - E_X[\nu^f])\end{aligned}$$

We use estimates obtained using country fixed effects, to estimate the effect of a sudden disappearance of polygyny. Using the individual regressions, we can display the different channels through which polygyny affects fertility. From the regressions using regional average, we can compute the macroeconomic effect of polygyny on fertility directly (see table 8). We present in the following table 11 those macroeconomic estimates using either the OLS or the IV multipliers.

According to our results, if polygyny would not have been in place in Benin for instance, the total number of children ever born in average per women would have been inferior from about 0.24 to 0.69 children.

Table 11: Macro effects of polygyny on total fertility

Regressions		Individual (tables 5 & 6)			Regional	Regional
Effect	Direct	via marriage	via remarriage	Total	Total	Total
Method	OLS	Probit	Probit	-	OLS	IV
Benin	-0.21	-0.03	-0.06	-0.30	-0.24	-0.69
Cameroon	-0.11	-0.02	-0.03	-0.16	-0.15	-0.44
Ivory Coast	-0.14	-0.04	-0.04	-0.22	-0.17	-0.50
Ghana	-0.12	-0.02	-0.02	-0.16	-0.12	-0.36
Madagascar	-0.01	0.00	0.00	-0.01	-0.02	-0.05
Malawi	-0.09	-0.01	-0.01	-0.11	-0.09	-0.26
Nigeria	-0.16	-0.04	-0.04	-0.24	-0.17	-0.50
Rwanda	-0.05	-0.02	-0.01	-0.08	-0.06	-0.17
Senegal	-0.19	-0.04	-0.07	-0.30	-0.22	-0.63
Tanzania	-0.12	-0.02	-0.03	-0.17	-0.14	-0.39
Uganda	-0.14	-0.03	-0.03	-0.20	-0.15	-0.44
Zimbabwe	-0.07	-0.01	-0.01	-0.09	-0.08	-0.23

Fertility is here the average number of children ever born per women.

7 Discussion and conclusion

In this paper, we brought forward two findings. Our first result is that women in polygamous union have a lower fertility rate than women in monogamous union. Although polygyny tends to increase the average duration of union and therefore the exposition to the risk of having a baby, most of the studies, as Pison (1986) and Pebley and Mbugua (1989) confirm that the individual fecundity of women in polygamous union is lower than that of those in monogamous union. There are many channels through which polygyny may decrease the individual fecundity of married women. First, man is less available as he has to split his times with each of his wife. However, Borgerhoff Mulder (1989) finds that this effect does not seem to play in Kipsigis family. Secund, a selection effect may play as a man may take an additional wife if he has not the number of children he wants with his first wife. However, Timaeus and Reynar (1998) find that women without children are in excessive numbers in polygamous unions. A third channel is that the age of the husband could have a significative impact on fecundity. As often in polygamous unions, young women tend to be married with older men and the declining fertility of aged men may affect the couple fecundity. Garenne and van de Walle (1989) find that the age of the husband has a negative and significant effect on fecundity. Lardoux and van de Walle (2003) find that the probability to give birth decreases significantly when the age of the husband increases. However, while this effect is clear after age 60 it may vary with the rank of the women and does not appear to be very strong. They also confirm that the fecundity of women in polygamous union is lower. The second result is the positive correlations at the regional (macroeconomic) level between polygyny and fertility. The link is likely to be causal between polygyny and fertility from our analysis. We find that, a rise in the share of women living within a polygamous household both increases average fertility and nuptials of the region. As women in a union have far more children that single ones, polygyny may increase the overall fertility rate. The impact of polygyny on the average fertility rate is therefore the sum of three contradictory effects: polygyny reduces individual fecundity of married women but increases average fertility both directly and through the nuptial effect. Our simulations show that, this effect is significant in the overall fertility rate. For instance, it increases the fertility rate up to 0.69 in Benin and 0.63 in Senegal. In the long run, polygyny is crucial to explain the population growth rate. This effect should be taken into account. The Sub-saharan African countries are at the doorstep of a new promising period of their evolution. In the next decades, if fertility rates decline as expected, they will experiment a demographic dividend with the reversal of the demographic dependence ratio. Such event will bring about an opportunity for the emergence of sustained economic growth (see Galor (2005)). Moreover, polygamous nature of families have effects on education investment in the family and children achievements. Lambert and Behaghel (2011) find that polygamy tends to have a negative effect on children's education, even for educated women. While the polygyny rate reaches 40% in certain countries, these effects may be determinant in Nation's future.

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Appendix

A) Data information

Table A : Sample of studied countries	
Countries where polygyny...	
...has been legalized	Malawi (2004)
...has been abolished	Benin (2004), Burundi (1993), Ivory Coast (1964), Uganda (2003)
...is legal	Cameroon, Senegal, Ghana, Rwanda
...is unlawful	Madagascar

Years of legal change are indicated between brackets.

Table B: DHS surveys used

	DHS II	DHS III	DHS IV	DHS V
Benin		1996	2001	2006
Cameroon	1991	1998	2004	
Cote d'Ivoire		1994, 1998		2005
Ghana		1993	1998, 2003	2008
Madgascar	1992	1997	2003	
Malawi	1992		2000, 2004	2008
Nigeria	1990		1999, 2003	2008
Rwanda	1992		2000	2005
Senegal	1992	1997	2005	
Tanzania	1991	1996	1999, 2003, 2004	2005, 2007
Uganda		1995	2000	2006
Zimbabwe		1994	1999	2005

B) Building individual infant mortality rate

B.1) Assumptions and framework

The main assumption is that the probability of death of a child does not depend on her rank into the brotherhood. Infant mortality is therefore assumed to be independent of the number of children a woman has[25].

One assumes that the infant's probability of death of a woman i can be decomposed into two components, a function $\mu(\bullet)$ of the observable individual characteristics X^i and an additional idiosyncratic probability z^i. To estimate the idiosyncratic component, we will use k^i and n^i, respectively the number of children dead[26] and the total children the woman had.

$$\mu^i = \mu(X^i) + z_i \tag{B.1}$$

[25] This assumption is probably a crude approximation as the first pregnancies in a woman's life, especially if they went wrong, are likely to induce complications during following ones.

[26] Here we call "dead" a child who did not survive beyond his first anniversary.

If we consider now the observed value of the mortality rate for a woman, it can only take discrete values which depends on n.

$$m^i = \frac{k^i}{n^i} \in \left\{\frac{p}{n^i}\right\}_{p=0}^{n^i} \tag{B.2}$$

Thus, the actual value of the mortality rate at the individual level depends on the number of children:

$$m^i = m(\mu^i, n) \tag{B.3}$$

Indeed if we consider the conditional probability of m^i given n and μ^i:

$$\mathcal{P}\left(\frac{k}{n}|n, \mu^i\right) = \binom{n}{k}(\mu^i)^k(1-\mu^i)^{n-k} \tag{B.4}$$

We can inverse this and compute the probability μ^i from m^i using the Bayes formula:

$$\mathcal{P}\left(\mu^i|n, m^i\right) = \frac{\mathcal{P}\left(m^i|n, \mu^i\right) \times \mathcal{P}(\mu^i)}{\int P(m^i|n, \mu^i) \times \mathcal{P}(\mu^i)} \tag{B.5}$$

The infant mortality's probability is to be computed at the individual level using Bayesian methods. To do so, we should first define a prior distribution of the idiosyncratic component z^i.

B.2) Using beta distributions to model priors

A natural candidate for the prior distribution is the uniform one. Assuming that $\mu^i \sim \mathcal{U}([0,1])$ allows calculating very easily the posterior distribution.

$$\mathcal{P}\left(\mu^i|n, m^i\right) = \frac{\binom{n}{k}(\mu^i)^k(1-\mu^i)^{n-k}}{\binom{n}{k}\int_0^1 x^k(1-x^i)^{n-k}dx} = \frac{(\mu^i)^k(1-\mu^i)^{n-k}}{\frac{(n-k))k!}{(n+1)!}} \tag{B.6}$$

We can therefore deduce the conditional expectancy for μ^i:

$$E\Big[\mu^i|n, m^i\Big] = \frac{(n+1)!}{(n-k)!k!}\int_0^1 x^{k+1}(1-x)^{n-k}dx = \frac{k+1}{n+2} \underset{n\to\infty}{\to} \mu^i \tag{B.7}$$

With such a "blind" prior, the empirical value converges to the actual probability for a very high number of children. But as the expectancy of the prior is arbitrary high equalling $\frac{1}{2}$, the estimates of μ^i for women with few children are largely biased upward. For instance a woman whose only child died as a mortality estimated to $\frac{2}{3}$, which is not plausible.

As $\mu^i \in [0,1]$ another convenient prior distribution is the beta distribution, $\mu^i \sim \mathcal{B}(\alpha, \beta)$.

Let us assume that the idiosyncratic component z is such that the resulting probability μ^i is distributed according to a beta distribution on [0,1], where Γ is the Gamma function[27].

$$\mathcal{P}(\mu^i) = g(\mu^i) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(\mu^i)^{\alpha-1}(1-\mu^i)^{\beta-1} \tag{B.8}$$

[27] $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$, $\Gamma(n) = (n-1)!$ if n is an integer

Interestingly, the parameter α and β can be set to replicate the expectancy and the variance of any prior distribution on [0,1]:

$$E[\mu^i] = \frac{\alpha}{\alpha+\beta} \ , \ V[\mu^i] = E[\mu^i] \times \frac{\beta}{(\alpha+\beta)(1+\alpha+\beta)} \tag{B.9}$$

$$\alpha = \frac{E^2(1-E)}{V} - E \ , \ \beta = \frac{\alpha(1-E)}{E} \tag{B.10}$$

The posterior probability of μ^i for k and n given remains conveniently a beta distribution:

$$\mathcal{P}(\mu^i|k,n) = \frac{(\mu^i)^{k+\alpha-1}(1-\mu^i)^{n-k+\beta-1}}{\int_0^1 x^{k+\alpha-1}(1-x^i)^{n-k+\beta-1}dx} \Rightarrow \mu^i \sim \mathcal{B}(k+\alpha, n-k+\beta) \tag{B.11}$$

Therefore, we can deduce the posterior expectancy of the sought probability μ^i using the proprieties of the Beta distribution:

$$E\Big[\mu^i|k,n\Big] = E\Big[\mathcal{B}(k+\alpha, n-k+\beta)\Big] = \frac{k+\alpha}{n+\alpha+\beta} \tag{B.12}$$

The posterior expectancy can be rewritten as a linear combination of the prior expectancy $E^0[\mu^i]$ and the observable mortality rate m^i. The weights depends of the expectancy and the variance of the prior distribution and the number of children.

$$E\Big[\mu^i|m^i,n\Big] = m^i \frac{n}{n+w_{E,V}} + E^0[\mu^i]\frac{w_{E,V}}{n+w_{E,V}} \ , \ w_{E,V} = \frac{E^0(1-E^0)}{V^0} - 1 \tag{B.13}$$

As expected, the Bayesian estimates relies more on the prior for the women with few children. Also, the more accurate the prior distribution is[28], the more the prior distribution matters for the posterior estimates.

B.3) Empirical estimation of the individual probability of infant mortality

We start by calculating for each region and area (rural or urban) the average mortality rate for babies born into a specific period, $mort_t^r$. We distinguish first the period 1960-1979[29] and each five years span between 1980 and 2010. The indicator $mort_t^r$ is calculated as the ratio between the total number of infants dead within an area and a given period and the total number of children born in the same area and during the same period. This indicator attends to measure the external factors[30] influencing infant mortality. We introduce it in a probit model to estimate E^0. We regress, for each country separately, the *actual* mortality rate m^i, see equation (B.14). We introduce also the year during which the woman was pregnant for the first time t_{p1}, the age a_{p1} and squared age at which the woman was pregnant for the first time and the mother's years of schooling h^i.

$$\mathcal{P}(m^i) = \Phi\left(\mu_0 + \mu^t t_{p1} + \mu^a a_{p1} + \mu^{aa} a_{p1}^2 + \mu^h h^i + \mu^r mort_t^r\right) \tag{B.14}$$

The expectancy E^0 is then set (B.15):

$$E^0 \equiv \mathcal{P}\Big(m^i|t_{p1}, a_{p1}, h^i, mort_t^r\Big) \tag{B.15}$$

[28] that is the lower the variance V^0 of the prior distribution is.

[29] Because all DHS surveys only focussed on women less than 50, only a few children are born before 1980.

[30] Which are not related to the characteristics of the parents.

It is difficult to estimate V^0 for such a regression. We will assume therefore that (B.16)[31]:

$$V^0 \equiv (E^0)^2 \tag{B.16}$$

C) Framework to simulate the macro effect of polygyny

To calculate the net effect of polygyny, let us consider the women with the vector of characteristics X, whose marital status s can be never married (n), currently married or in a relationship (m) or single but formerly married (f). Let us designate ν_X^s the fertility rate of the woman with characteristics X and marital status s. Let us also denote p_X^s the probability that the marital status of such a woman be s. The average fertility rate of a woman with characteristics X, ν_X is therefore:

$$\nu_X = \nu_X^n p_X^n + \nu_X^f(1-p_X^n)p_X^f + \nu_X^m(1-p_X^n)(1-p_X^f) \tag{C.1}$$

Let us denote π the local share of women married to a polygamous man. Polygyny is not to influence the fertility rate of unmarried woman directly[32]. The marginal effect of the incidence of polygyny on fertility is therefore:

$$\begin{aligned}\frac{d\nu_X}{d\pi} &= (1-p_X^n)\left(\frac{d\nu_X^f}{d\pi}p_X^f + \frac{d\nu_X^m}{d\pi}(1-p_X^f)\right) + \frac{dp_X^n}{d\pi}\left(\nu_X^n - \nu_X^m - p_X^f(\nu_X^f - \nu_X^m)\right) \\ &+\frac{dp_X^f}{d\pi}(\nu_X^f - \nu_X^m)(1-p_X^n)\end{aligned} \tag{C.2}$$

The first term measures the direct effect of polygyny on fertility. The second term represents the effect of polygyny on fertility via the raise in the nuptial rate, whereas the third term stands for the effect of polygyny on the probability of remarriage. Unfortunately no information is available in the DHS surveys to determine whether a formerly married woman was bound to a man with several spouses or not. One makes the assumption that polygyny has the same effect on fertility for currently and formerly married women.

$$\frac{d\nu^f}{d\pi} \approx \frac{d\nu^m}{d\pi} \tag{C.3}$$

Moreover, thanks to the linear econometric specification chosen to estimate the microeconomics effect of polygyny on married women, one has:

$$\frac{d\nu^m}{d\pi} = \zeta \tag{C.4}$$

As the probability p_n is modelled with a probit model, one can write:

$$\frac{dp_X^n}{d\pi} = \zeta_n \Phi'(X^i\beta_n + \zeta_n\pi^k + Y^k\theta_n) \tag{C.5}$$

[31] In practice, this convention does not modify much the results.

[32] That is $\frac{d\nu_X^n}{d\pi} = 0$

This expression can be simplified by evaluating the marginal probit function at the mean, with $s = \{n, f\}$

$$\frac{dp_X^s}{d\pi} \approx \zeta_s E\Big[\Phi'(X^i\beta_s + \zeta_s\pi^k + Y^k\theta_s)\Big] \equiv \frac{dp^s}{d\pi} \tag{C.6}$$

These allows to simplify the previous expression.

$$\frac{d\nu_X}{d\pi} \approx (1-p_X^n)\frac{d\nu^m}{d\pi} - \frac{dp^n}{d\pi}\Big(\nu_X^n - \nu_X^m - p_X^f(\nu_X^f - \nu_X^m)\Big) - \frac{dp^f}{d\pi}(\nu_X^f - \nu_X^m)(1-p_X^n) \tag{C.7}$$

One can use this equation to evaluate the overall impact of polygyny on aggregate fertility by integrating the previous equation over all the characteristics X. Let us note ϕ_X the frequency of women with the vector of characteristics X in the overall population. Let us note $J = M \cap N \cap F$ the overall population of women, whether they are married ($\in M$), never been married ($\in N$) or were formerly married ($\in F$). Asymptotically, the expectancy over characteristics equals the average over the population:

$$\frac{dE[\nu]}{d\pi} = E\left[\frac{d\nu}{d\pi}\right] = \frac{1}{\#J}\sum_{j\in J}\frac{d\nu^j}{d\pi} = \sum_X \frac{d\nu_X}{d\pi}\phi_X \tag{C.8}$$

If the estimator of p_n is unbiased, one has also asymptotically the following equality, which allows to calculate easily the expectancy of p_n.

$$\sum_X p_X^i\phi_X = \frac{\#I}{\#J} \equiv p^i \tag{C.9}$$

However, the aggregate fertility rate among the group i, ν^i should be calculated using the projection of v_X^i on the overall population and not on using average fertility among only among i.

$$\sum_X v_X^i\phi_X = \frac{1}{\#J}\sum_{j\in J}\nu_j^i \neq \frac{1}{\#I}\sum_{j\in I}\nu_j \tag{C.10}$$

Let us note $E_X[\nu^i] \equiv \sum_X \nu_X^i\phi_X$ to ease the calculations. The sought expectancy (13) becomes:

$$\begin{aligned} E\left[\frac{d\nu}{d\pi}\right] &\approx (1-p^n)\frac{d\nu^m}{d\pi} - \frac{dp^n}{d\pi}\Big(E_X[\nu^m] - E_X[\nu^n] - p^f(E_X[\nu^m] - E_X[\nu^f])\Big) \\ &\quad - \frac{dp^f}{d\pi}(1-p^n)(E_X[\nu^m] - E_X[\nu^f]) \end{aligned} \tag{C.11}$$