Taxation, Income Redistribution and Models of the Household

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Abstract

This paper compares the properties of optimal piecewise linear tax systems based on joint and individual incomes respectively. A key aspect of the analysis is the distinction between second earner wage differences and variation in productivity in household production as determinants of across-household heterogeneity in second earner labour supply. This is work in progress. Please do not quote or cite without referring back to the authors

1 Introduction

This paper seeks to bring the analysis of the optimal taxation of two-earner households closer to reality in two important respects. First, we analyse the optimal choice of the parameters of a piecewise linear tax system, as opposed both to a general nonlinear tax system, based upon the mechanism design approach of Mirrlees (1971), and to the two-parameter linear tax system studied by Sheshinski (1972). The reason for this is simply that real tax systems are almost universally of the piecewise linear kind, yet there has been very little analysis of their optimal structure,[1] and none at all of the two-earner household case.

Second, we base the tax analysis on a model of the household that conforms to the data on the time use of family households consisting two adults, at least one of whom is in full time employment, together with children. In such households, household production, particularly in the form of child care, is a major form of time use, and, we argue, this has important implications for the nature of the across-household relationships among second earner labour supply,

[1] The main references are Apps, Long and Rees (2011), Slemrod et al (1994) and Sheshinski (1989). For further discussion of the literature see the first of these.

household income and utility possibilities, that are of fundamental importance in the design of tax systems. Again, this paper is the first to consider this issue in the context of piecewise linear taxation.

The two central issues in the design of such a tax system for two-earner households are the choice of tax base, individual or joint income, and of a rate scale, in particular whether the marginal tax rates applying to successive income brackets should be strictly increasing, or whether over at least some income ranges they should be decreasing. In Apps, Long and Rees (2009) we refer to these as the "convex" and "nonconvex" cases respectively, to describe the types of budget sets in the space of gross income-net income/consumption to which they give rise. They could also be described as "progressive" and "regressive", as long as it is understood that these terms refer to the marginal rather than average tax rate.[2] There we show that which of these structures is likely to be optimal depends closely on the distribution of wage rates, and that given the actual empirical distributions, convex systems are very likely to yield welfare-superior results.

In this paper, for our purposes it is sufficient to focus on the convex case, which is analytically simpler to deal with. We discuss two issues which, as suggested above, have not to date been considered in the context of piecewise linear tax systems. The first is the comparison of joint with individual taxation, the second is the issue of the effects of the existence of household production on optimal taxation, given its implications for the relationship between a household's utility possibilities and its labour market income. We carry out the analysis in two steps.

First we find the optimal piecewise linear tax systems for the case of joint and individual taxation respectively, making the standard assumption that each individual's time is divided between market work and leisure, the direct consumption of one's own time. It is important to note that by "individual" taxation we mean the case in which the two earners' incomes are taxed separately but according to the same tax schedule. This is in contrast to what we call "selective taxation", under which two separate optimal tax schedules are found for primary and second earners respectively. Up until now, this problem has only been considered in the case of linear taxation.[3] One reason for our focus on individual rather than selective taxation is that the distinction between individual and selective taxation does not arise in the linear case, while it is not difficult to see in general terms what the solution of the piecewise linear selective tax problem would be by analogy with the linear tax problem. A further reason is that in practice, piecewise linear tax systems that are not joint are of this kind, possibly because, for constitutional and political reasons, it would not be

[2] Since of course a tax system with decreasing marginal rates could still be average-rate progressive.

[3] See Boskin and Sheshinski (1986) and Apps and Rees (). Kleven, Kreiner and Saez (2009) use a Mirrlees optimal tax framework to investigate the way in which the tax function defined on the primary earner's income should depend on the second earner's decision whether to work full time in the market for a wage that is the same for all second earners, or not to work in the market at all. This "optimal implicit participation tax problem" is a somewhat different issue to that analysed here.

feasible to introduce differentiated tax systems for primary and second earners.

The second step is then to reconstruct the household model to incorporate time spent in household production, which we refer to as "child care", and to analyse the effects of this on the optimal tax analysis. As Sandmo (1990) showed,[4] it is possible to formulate the optimal tax problem in such a way that the introduction of household production has no apparent effect on the optimal tax *conditions.* We show that this also applies here. Nevertheless, as we also show, the properties of the optimal tax sytem and the comparison between joint and individual taxation are profoundly affected. The analysis without household production shows that there are gains in both equity and efficiency in moving from optimal joint to optimal individual taxation. In the presence of untaxed household production, there are further gains from such a move, arising out of the progressivity of the piecewise linear tax system, and these gains were not capable of being identified in the linear tax analysis. In other words, the analysis of piecewise linear tax systems in the presence of household production strengthens the case for individual taxation, even when not selective, still further.

The crux of the issue is that in the presence of untaxed household production, the relationship between a household utility possibilities and total labour income need no longer be monotonic.

2 Models

In this section we set out the two household models. The first is the conventional model containing two adults who allocate time between market work and leisure.[5] The second is the model of the two-parent family in which the adults allocate time to market work and household production, which we designate as child care, rather than leisure.[6]

In each model there is a composite market consumption good, x. Individuals face given gross wage rates w, representing their productivities in a linear aggregate production technology that produces x, and have earnings y from their labour supply.

The two adults in a household are designated as primary and second earners respectively, with the former receiving a strictly[7] higher wage than the latter.[8]

[4] Sandmo (1990) was the first to analyse optimal linear income taxation in the presence of household production, albeit for single-person households. Kleven, Richter and Sørenson () extended his model to show that the well-known Corlett-Hague result must be refomulated, since it does not hold as it stands in the presence of household production, even for single person households. It is perhaps also worth pointing out that this is also true for the equally well-known Atkinson-Stiglitz Theorem, since the separability between leisure and consumption no longer gives the result if household goods are Hicksian complements or substitutes to market goods.

[5] As for example in Boskin and Sheshinski (1986).

[6] Nothing would be gained by retaining leisure as a form of time use in this model, so in the interests of notational simplicity we dispense with it.

[7] This is simply to avoid ambiguity.

[8] This almost follows from the definition of "primary" and "second" earners, according to

The tax system pays households a lump sum[9] and taxes its labour earnings according to a two-bracket piecewise linear rate schedule, which determines how the lump sum payment is funded. We consider the implications first, of taxing their joint income, and secondly, of taxing them individually but under the same schedule. In each case we characterise the optimal tax schedule, and compare the resulting welfare level, tax rates and the extent of redistribution. We carry out this analysis for both household models, in order to examine the implications for the comparison of the two types of tax system of a change in the cause of across-household heterogeneity in second earner labour suppliy and income - wage rates as in the first model vs. productivity in household production as in the second.

2.1 Model 1

By allowing for two-earner households, this model takes one step towards reality, as compared to the standard models used in the economics of taxation. The second step, the incorporation of household production in the form of child care, is left for Model 2.

There are P types of primary and S types of second earners, defined by their wage rates, with $w_1 \in \{w_1^1, w_1^2, ..., w_1^P\}$ and $w_2 \in \{w_2^1, w_2^2..., w_2^S\}$, $w_2^1 < w_1^1$, $w_2^S < w_1^P$ and in every household $w_2 < w_1$. Subject to this restriction, household type is then defined by the pair (w_1, w_2). Let h index these pairs as follows: choose $w_1 = w_1^1$, denote by $h = 1$ the household (w_1^1, w_2^1), and then let w_2 increase, numbering the households consecutively, until the largest second earner wage is reached such that $w_2 < w_1^1$. Call this household h'. Then take household (w_1^2, w_2^1) as $h' + 1$, and let w_2 increase, numbering the households consecutively, until the largest wage is reached such that $w_2 < w_1^2$, and so on. Household H will correspond to the wage pair (w_1^P, w_2^S). Thus we index the household wage pairs (w_{1h}, w_{2h}) lexicographically so that

$$h > h' \Leftrightarrow w_{1h} > w_{1h'} \text{ or } w_{1h} = w_{1h'} \text{ and } w_{2h} > w_{2h'} \ i = 1, 2, \quad h = 1, ..., H$$

This convention determines how household welfare,[10] labour supply and income will vary with h. Note that it does not imply that household income increases monotonically with h, since one household may have a higher primary wage than another but a sufficiently lower second wage that household income is lower.

There are two closely related reasons for basing the index h on wage rates:

- wage rates are exogenous whereas incomes are endogenous
- wage rates, rather than incomes as such, determine a household's utility possibility set

which the latter's income is by definition smaller. It simply rules out the possibility that the higher wage partner works sufficiently fewer hours that she has the lower income.

[9] Which coud be thought of as a child benefit, though here it does not vary with the number of children.

[10] Of course, only individuals, and not households, can have "welfare", but we will frequently use this term to refer to the set of feasible utility pairs that a household can enjoy.

The household's utility function[11] is

$$u_h = x_h - \sum_{i=1}^{2} \psi(y_{ih}, w_{ih}) \quad h = 1, ..., H \tag{1}$$

where the $\psi(.)$ are identical within and across households, strictly increasing and strictly convex in y_{ih} and possess the single-crossing property

$$\frac{\partial}{\partial w_{ih}}[\frac{\partial \psi}{\partial y_{ih}}] > 0 \quad i = 1, 2, \quad h = 1, ..., H \tag{2}$$

This says that the higher the wage type, the lower the marginal effort cost to i of achieving a given increase in labour earnings.[12] It implies that of two individuals facing the same marginal tax rate, the one with a higher gross wage rate will have the higher labour supply and earned income.

In fact, in this model household utility, labour supply and therefore income increase monotonically with wage rates. At a given primary earner wage, heterogeneity across households in second earner labour supply and income is then driven entirely by variation in the second earner wage, so that a household with low second earnings must have a low second wage. If it is assumed that the primary earner "shares" his income to ensure equal consumption shares, this kind of model underpins the equity argument for joint taxation or, equivalently, income splitting.

The utility function (1) is simple, but, since it is defined on total household consumptions, implies that we can say nothing about the intrahousehold utility distribution. Essentially, we are assuming that the household allocates its resources between its members in exactly the way that the "social planner" would wish it to.[13]

2.2 Model 2

The preceding model suffers from the limitation that each member of the household has a simple division of time between market work and leisure and as a result, given the assumption of identical utility functions, wage incomes are a good indicator of achieved utility levels. The power of income taxation in redistributing utility depends on the strength of the association between the marginal social utility of income and income and, as long as this is (negatively) monotonic, it seems incontrovertible that redistributing income from higher to lower income earners is progressive in its effects and will increase social welfare.

[11] The quasilinear and additively separable form assumed here, though special, is very convenient, since it eliminates income effects and greatly simplifies the presentation of the optimal tax formulas.

[12] This type of utility function is widely used in optimal tax theory and could be rationalised by assuming a standard strictly concave and increasing utility function defined on leisure, with labour supply given by the time endowment *minus* the time spent in consuming leisure. This is made more explicit in Section X below.

[13] For further discussion of this point, see Apps and Rees (2009) ch. 7.

However, the nature of the relationship between income and a household's utility possibilities becomes ambiguous when we take account of household production as a form of time use in two-earner households,[14] and this in turn has important implications for tax policy.

We now extend the household model in a simple and tractable way to take account of household production, which we will call child care, and analyse the implications for the type of tax analysis carried out above.

In addition to the market consumption good x the household now also consumes child care z, which is produced using parental time inputs c_i, $i = 1, 2$, according to the concave increasing production function

$$z = z(c_1, c_2; k) \tag{3}$$

Here, $k \in \{k^1, ..., k^Q\}$ is an exogenous productivity parameter that varies across households, and captures the idea that a household's productivity in producing child care[15] will depend on its given stock of human and physical capital, with $\partial z/\partial k > 0$.

The introduction of the productivity parameter k adds a further dimension to household type, which now depends on the triple (w_1, w_2, k). To keep things simple, we make the assumption of perfect assortative matching, $w_2 = \beta w_1$, $\beta \in (0, 1)$, and so a household's type can be characterised by a pair of values (w_1, k), with again $w_1 \in \{w_1^1, w_1^2, ..., w_1^P\}$. We can again define the household index $h = 1, ..., H$ by taking $(w_1^1, k^1), ..., (w_1^1, k^Q)$, then $(w_1^2, k^1), ..., (w_1^2, k^Q)$, and so on, so that household H is characterised by (w_1^P, k^Q), and has the highest wage rate(s) and productivity, and therefore the highest utility possibilities. Thus, in this model, at any given primary earner wage rate, across-household heterogeneity is driven by productivity variation rather than wage variation.

The key point here is that it cannot in general be assumed that increasing productivity increases second earner labour supply and household income.[16] The intuition is straightforward: an increase in productivity reduces the time required to produce a given amount of z, and therefore makes possible an increase in market labour supply, but also reduces its implicit price and therefore increases its demand, given that it is a normal good. Thus there are opposing effects acting on the second earner's labour supply. This implies that the relationship between household income and utility possibilities is no longer necessarily positive or monotonic, and therefore has an important effect on the interpretation of the results of an optimal tax analysis, as we show below.

The household utility function is now given by

$$u_h = x_h + \hat{u}(z_h) \quad h = 1, ..., H \tag{4}$$

The $\hat{u}(.)$ function, which treats child care as a household public good, is strictly increasing and strictly concave. So, in this model, child care replaces leisure as

[14] As we have previously argued. See Apps and Rees (1988), (1996), (2009).

[15] Which should be thought of as generally as embodying "child outcomes" and not simply as the time spent in child-minding.

[16] We have shown this formally in a number of related models of this type. For further discussion and references see Apps and Rees (2009).

the second good. For each individual, the time spent in market work and child care must sum to the total time endowment, normalised at 1, and so we have

$$c_{ih} + l_{ih} = 1 \quad i = 1, 2, \;\; h = 1, ..., H \tag{5}$$

where l_{ih} is market labour supply. Recalling that $y_{ih} = w_{ih} l_{ih}$, we can use the time constraint to eliminate the c_{ih} and rewrite $\hat{u}(.)$ as

$$\hat{u}[z(c_{1h}, c_{2h}; k_h)] \equiv \hat{u}[z(1-y_{1h}/w_{1h}, 1-y_{2h}/w_{2h}; k_h)] \equiv -\varphi(y_{1h}, y_{2h}; w_{1h}, w_{2h}, k_h) \tag{6}$$

It is straightforward to establish that the function $\varphi(.)$ possesses the same properties as $\psi(.)$ in the previous models. The productivity parameter k_h however introduces a fundamentally new set of considerations into the model, as we have just suggested.

Writing the household budget constraint as

$$x_h \leq \sum_{i=1}^{2} y_{ih} - T(y_{1h}, y_{2h}) \quad h = 1, ..., H \tag{7}$$

we see that we now have a model that is similar to the previous one, in that the "observable" variables, the consumptions and labour earnings, x_h and y_{ih}, have the same kinds of effects as in the previous model. As we show below,we can carry out the optimal tax analysis for both models at the same time and derive exactly the same expressions for the optimal tax parameters. The key difference lies in the interpretation of the results, and in their implications for the comparison of joint and individual tax systems. We shall show that the existence of (unobservable and non-taxable) household production with varying productivities across households further strengthens the case for individual as opposed to joint progressive income taxation.

2.3 Tax Functions

In both models the household budget constraint is:

$$x_h \leq \sum_{i=1}^{2} y_{ih} - T(y_{1h}, y_{2h}) \quad h = 1, ..., H \tag{8}$$

with tax functions $T(.)$ specified as follows.

Joint Taxation:

There is a two-bracket piecewise linear tax on total household labour earnings, the parameters of which are $(\alpha, \tau_1, \tau_2, \eta)$, where α is a uniform lump sum paid to every *household*, $\tau_1$and τ_2 are the marginal tax rates in the lower and upper brackets of the tax schedules, and η is the value of joint earnings defining the bracket limit. Thus the household labour earnings tax function $T(y_{1h}^1, y_{2h}^1) \equiv T(y_h)$, with $y_h = \sum_{i=1}^{2} y_{ih}$, is defined by:

$$T(y_h) = -\alpha + \tau_1 y_h \quad y_h \leq \eta \tag{9}$$

$$T(y_h) = -\alpha + \tau_2 y_h + (\tau_1 - \tau_2)\eta \quad y_h > \eta \;\; h = 1, ..., H \tag{10}$$

Individual Taxation:

There is a two-bracket piecewise linear tax system now applied to individual labour earnings, the parameters of which are (a, t_1, t_2, y), where a is again a uniform lump sum paid to every household, $t_1$and t_2 are the marginal tax rates in the lower and upper brackets, and y is the value of *individual* earnings defining the bracket. Thus the individual labour earnings tax function $T(y_{ih})$ is defined by:

$$T(y_{ih}) = t_1 y_{ih} \quad y_{ih} \leq y \tag{11}$$

$$T(y_{ih}) = t_2 y_{ih} + (t_1 - t_2)y \quad y_{ih} > y \;\; h = 1, ..., H \tag{12}$$

and, with a small abuse of notation, the household tax function is $T(y_{1h}, y_{2h}) \equiv -a + \sum_{i=1}^{2} T(y_{ih})$.

Throughout this paper, as mentioned in the Introduction, we assume that we have what we call the convex case, in which at the tax optima $\tau_1 < \tau_2$ and $t_1 < t_2$. Every household faces the same convex budget constraint in the (y_h, x_h)- and (y_{ih}, x_h)-planes respectively.

3 Household Allocations

In this section we analyse the household's choice of consumption and wage earnings under each of the two alternative tax systems, first joint and then individual taxation. We do so for Model 1, and then show that exactly the same formal results are derived for Model 2.

The main aim is to derive the indirect utility functions giving household welfare as a function of the tax parameters in each case. The basic analysis for the two tax systems, joint and individual, is essentially quite similar. The main difference is that when we partition the set of households into subsets according to the marginal tax rate each individual is facing, in the case of joint taxation we have only three subsets, while in the case of individual taxation (given that each faces the same tax schedule) there are six. The former case excludes the possibility that individuals in the same household can face different marginal rates, the latter allows it.

3.1 Joint Taxation

A household solves the problem

$$\max_{x_h, y_{ih}} u_h = x_h - \sum_{i=1}^{2} \psi(y_{ih}, w_{ih}) \tag{13}$$

subject to a budget constraint determined by the tax system. We consider three cases which provide the results we require - the partial derivatives of the household's indirect utility function with respect to the tax parameters. We write below the constraints for each of these cases together with these derivatives.

Case 1. The household is at the optimum in the interior of the lower tax bracket. It therefore faces the constraint:

$$x_h = \alpha + (1 - \tau_1)\sum_i y_{ih} \quad (14)$$

and the first order conditions imply:

$$\frac{\partial \psi}{\partial y_{1h}} = \frac{\partial \psi}{\partial y_{2h}} = 1 - \tau_1 \quad (15)$$

giving the earnings supply functions $y_{ih}(\tau_1, w_{ih})$. The properties of the functions $\psi_i(.)$ imply

$$\frac{\partial y_{ih}(t_1, w_{ih})}{\partial \tau_1} < 0, \; \frac{\partial y_{ih}(t_1, w_{ih})}{\partial w_{ih}} > 0 \quad i = 1, 2, \quad h = 1, ..., H \quad (16)$$

where, note, the first of these is a compensated derivative. We write the household indirect utility function as $v_h(\alpha, \tau_1)$, with, by the Envelope Theorem,

$$\frac{\partial v_h}{\partial \alpha} = 1; \quad \frac{\partial v_h}{\partial \tau_1} = -y_h = -\sum_i y_{ih}(\tau_1, w_{ih}) \quad i = 1, 2 \quad (17)$$

Case 2. The household is effectively constrained at the bracket limit η, in the sense that it chooses $y_h = \eta$, but would prefer to increase its labour supply and earnings if it would be taxed at the rate τ_1, but not if it would be taxed at the rate τ_2. We formulate its allocation problem by adding the constraint $y_h \leq \eta$ to its optimisation problem, noting that this will be binding at the optimum.[17] We write its indirect utility function as $v_h(\alpha, \tau_1, \eta)$, with, by the Envelope Theorem,

$$\frac{\partial v_h}{\partial \alpha} = 1; \quad \frac{\partial v_h}{\partial \tau_1} = -\eta; \quad \frac{\partial v_h}{\partial \eta} = (1 - \tau_1) - \frac{\partial \psi}{\partial y_{ih}} \geq 0 \quad (18)$$

Intuitively, the idea of the expression for $\partial v_h / \partial \eta$ is that a small relaxation of the constraint would increase consumption and utility at the rate $(1 - \tau_1)$, which (weakly) exceeds for each individual the marginal cost of effort $\partial \psi / \partial y_{1h} = \partial \psi / \partial y_{2h}$. In diagrammatic terms, the household is at the kink in its budget constraint which exists at the bracket limit η. The term is zero only if i's marginal rate of substitution happens to be $(1 - \tau_1)$ at the kink.

Case 3. The household is in equilibrium in the interior of the upper income bracket. We therefore replace the previous budget constraint by

$$x_h \leq \alpha + (1 - \tau_2) y_h + (\tau_2 - \tau_1)\eta \quad (19)$$

and the first order conditions imply

$$\frac{\partial \psi}{\partial y_{1h}} = \frac{\partial \psi}{\partial y_{2h}} = 1 - \tau_2 \quad (20)$$

[17] Case 1 can be thought of as the case in which this constraint is non-binding.

giving the earnings supply functions $y_{ih}(\tau_2, w_{ih})$. The properties of the functions $\psi(.)$ imply

$$\frac{\partial y_{ih}(\tau_2, w_{ih})}{\partial \tau_2} < 0, \quad \frac{\partial y_{ih}(\tau_2, w_{ih})}{\partial w_{ih}} > 0 \quad i = 1, 2, \quad h = 1, ..., H \tag{21}$$

Writing the indirect utility function as $v_h(\alpha, \tau_1, \tau_2, \eta)$ we now obtain

$$\frac{\partial v_h}{\partial \alpha} = 1; \quad \frac{\partial v_h}{\partial \tau_1} = -\eta; \quad \frac{\partial v_h}{\partial \tau_2} = -(y_h - \eta); \quad \frac{\partial v_h}{\partial \eta} = \tau_2 - \tau_1 > 0 \tag{22}$$

It is useful to have the following notation. Let $\{\mathcal{H}_0, \mathcal{H}_1, \mathcal{H}_2\}$ denote a partition of the index set $\{1, 2, ..., H\}$ defined as follows:

$$\mathcal{H}_0 = \{\ h \ \mid y_h < \eta\} \tag{23}$$

$$\mathcal{H}_1 = \{\ h \mid y_h = \eta)\} \tag{24}$$

$$\mathcal{H}_2 = \{\ h \ \mid y_h > \eta\} \tag{25}$$

where y_h is the household's optimal income under the given tax structure. In all of what follows we assume that we are dealing with tax systems in which all these subsets are non-empty. Clearly total household gross and net income and therefore, in this model, household utility are increasing as we move from $\mathcal{H}_0$ to $\mathcal{H}_1$ to $\mathcal{H}_2$, though these may not increase monotonically with h *within* any of these subsets, as pointed out earlier.

3.2 Individual Taxation

Given a piecewise linear tax schedule with parameters (a, t_1, t_2, y), but in which the individuals in the household are free to choose their individually optimal earnings value, there are six types of possible household optimum and therefore six possible subsets into which we can partition the set of households:

$$H_0 = \{\ h \ \mid y_{ih} < y,\ i = 1, 2\} \tag{26}$$

$$H_1 = \{\ h \ \mid y_{2h} < y,\ y_{1h} = y\} \tag{27}$$

$$H_2 = \{\ h \ \mid y_{ih} = y,\ i = 1, 2\} \tag{28}$$

$$H_3 = \{\ h \ \mid y_{2h} < y,\ y_{1h} > y\} \tag{29}$$

$$H_4 = \{\ h \ \mid y_{2h} = y,\ y_{1h} > y\} \tag{30}$$

$$H_5 = \{\ h \ \mid y_{ih} > y,\ i = 1, 2\} \tag{31}$$

Given each subset, it is straightforward to derive the earnings supply and indirect utility functions just as we did in the previous subsection. The obvious difference is that only in subsets H_0 and H_5, where the individuals in the household face the same marginal tax rates, will the derivatives $\partial\psi/\partial y_{ih}$ be equalised. In all other cases they will not in general be the same. We draw directly on the results for the derivatives of the indirect utility function presented in the previous subsection when we carry out the optimal tax analysis for individual taxation in Section 5 below.

Contrasting the partition defined by (23)-(25) in this case with that in (26)-(31) for the joint taxation case makes clear the essential difference between joint and individual taxation. The latter implies a much finer partition into subsets reflecting likely differences in responsiveness of *individual* earnings (labour supply) decisions to tax rates, which is the source of the efficiency gains brought out by the analysis of optimal linear taxation[18] and tax reform[19] Lower income second earners, who empirically have much higher labour supply elasticities, are sorted into the lower tax bracket.

The equity effects of this finer matching of individuals with tax brackets are less easy to establish. In the absence of lump sum compensation, households with very low second earner labour supplies tend to be made worse off by a switch from joint to individual taxation, since the tax burden on primary earners is increased while that on second earners is reduced.[20] The simulation analysis we present later in Section 6 shows that the overall equity effects of this change in tax structure are very strongly dependent on the shape of the earnings distribution, and that, for realistic assumptions on the form of this distribution and reasonable specifications of the social welfare function, these equity effects are also positive.

As suggested earlier, if we now replace the function $\psi(.)$ with $\varphi(.)$ in the above analysis nothing would appear to change, the household solution possibilities and general forms of the indirect utility functions would *appear* to be the same. Underlying them however is a fundamentally different model of the household and this will, as we shall see, affect the interpretation of the results in an important way.

4 Optimal Tax Analysis: Model 1

4.1 Joint Taxation

The planner solves

$$\max_{\alpha,\tau_1,\tau_2,\eta} \sum_{h=1}^{H} \phi_h S(v_h) \tag{32}$$

subject to the public sector budget constraint[21]

$$\sum_{h\in\mathcal{H}_0\cup\mathcal{H}_1} \phi_h \tau_1 y_h + \sum_{h\in\mathcal{H}_2} \phi_h[\tau_2 y_h + (\tau_1 - \tau_2)\eta] \geq \alpha \tag{33}$$

where ϕ_h is the proportion of households of type $h = 1, 2, ..., H$, and $S(.)$ is a strictly concave and increasing function expressing the planner's preferences over

[18] See Boskin and Sheshinski (), Apps and Rees ().

[19] See Apps and Rees ().

[20] For a thorough analysis of this in the tax reform context see Apps and Rees (), (), ().

[21] We assume the aim of taxation is purely redistributive. Adding a non-zero revenue requirement would make no difference to the results.

household utilities. From the first order conditions characterising the optimal tax parameters[22] we can derive:

Proposition 1: The optimal tax parameters satisfy the conditions:

$$\sum_{h=1}^{H} \phi_h(\sigma_h - 1) = 0 \tag{34}$$

$$\tau_1 = \frac{\sum_{\mathcal{H}_0} \phi_h(\sigma_h - 1)y_h^* + \eta \sum_{\mathcal{H}_1 \cup \mathcal{H}_2} \phi_h(\sigma_h - 1)}{\sum_{\mathcal{H}_0} \phi_h \partial y_h / \partial \tau_1} \tag{35}$$

$$\tau_2 = \frac{\sum_{\mathcal{H}_2} \phi_h(\sigma_h - 1)(y_h^* - \eta)}{\sum_{\mathcal{H}_2} \phi_h \partial y_h / \partial \tau_2} \tag{36}$$

$$\sum_{\mathcal{H}_1} \phi_h\{\sigma_h[(1-\tau_1) - \frac{\partial \psi}{\partial y_{ih}}] + \tau_1\} = -(\tau_2 - \tau_1) \sum_{\mathcal{H}_2} \phi_h(\sigma_h - 1) \tag{37}$$

where y_h^* denotes household income at the optimum and σ_h is the marginal social utility of income to household h.

Condition (34) is familiar from linear tax theory: the optimal lump sum α equalises the average of the marginal social utilities of household income, σ_h, in terms of the numeraire, with the marginal cost of one unit of the lump sum, which of course is 1. Denoting the shadow price of the government budget constraint by λ, $\sigma_h \equiv S'(v_h)/\lambda$, and so the concavity of $S(.)$ implies that σ_h falls with the utility level of the household. In the household model underlying this tax analysis, household utility increases with household income, and so the average value of σ_h falls as we move from $\mathcal{H}_0$ to $\mathcal{H}_1$ to $\mathcal{H}_2$. Since (34) implies that

$$\sum_{\mathcal{H}_0} \phi_h(\sigma_h - 1) = - \sum_{\mathcal{H}_1 \cup \mathcal{H}_2} \phi_h(\sigma_h - 1)$$

it can be shown that

$$\sum_{\mathcal{H}_0} \phi_h(\sigma_h - 1) > 0 > \sum_{\mathcal{H}_1 \cup \mathcal{H}_2} \phi_h(\sigma_h - 1)$$

The two conditions corresponding to the tax rates τ_1, τ_2, are analogous to those obtained in optimal linear tax theory. The denominators are the average compensated derivatives of earnings (labour supply) with respect to the tax rates, and so give a measure of the marginal deadweight loss of the tax rate at the optimum, the efficiency cost of the tax. The numerators give the equity effects. The two terms in the numerator of (35) correspond to the two ways in which the lower bracket tax rate affects the contributions households make to funding the lump sum payment α. Given their optimal earnings y_h^*, the first term aggregates over subset $\mathcal{H}_0$ the effect of a marginal tax rate change on welfare

[22] Of course, exactly which households will be in which subsets is determined at the optimum, and depends on the values of the tax parameters. The following discussion characterises the optimal solution *given* the allocation of households to subsets that obtains at this optimum.

net of its marginal contribution to tax revenue, all in terms of the numeraire. The second term reflects the fact that the lower bracket tax rate is effectively a lump sum tax on income earned by the two higher brackets, $\mathcal{H}_1$ and $\mathcal{H}_2$, since a change in this tax rate changes the tax they pay at a rate given by η.

Only the first of these two effects is of course present in the condition corresponding to the second tax rate. The portion of the income of the households in the higher tax bracket that is taxed at the rate τ_2 is $(y_h^* - \eta)$, and so this weights the effect on social welfare net of the effect on tax revenue. Note that, unlike the case of linear income taxation, these numerator terms are not covariances, since the mean of σ_h over each of the subsets in not 1. However, intuitively they can still be thought of as measures of the strength of the relationship between the marginal social utility of income and household incomes, which determines the effectiveness of the tax rate on *income* in redistributing *utility* across households.

It is interesting to rewrite this numerator term as

$$\sum_{\mathcal{H}_2} \phi_h(\sigma_h - 1)y_h^* - \eta \sum_{\mathcal{H}_2} \phi_h(\sigma_h - 1) \tag{38}$$

where the second term is seen to be the negative of the second term in the numerator of (35), net of the lump sum tax contribution of the subset $\mathcal{H}_1$. This suggests that the greater the contribution of the lump sum tax on upper bracket households arising from the tax rate τ_1, the smaller is the tax rate τ_2, and so the smaller is the distortionary effect on labour supplies in this bracket, other things being equal.

Condition (37), the condition on the bracket limit η, has the following interpretation. The left hand side represents the marginal social benefit of a slight relaxation of the bracket limit. This consists first of all of the gain to all those households who are effectively constrained at η, as discussed earlier. The first term in brackets on the left hand side is the net marginal benefit to these consumers, weighted by their marginal social utilities of income. The second term is the rate at which tax revenue increases given the increase in gross income resulting from the relaxation of the bracket limit. The right hand side gives the marginal social cost of the relaxation. Since $(\tau_2 - \tau_1) > 0$ by assumption, all households $h \in \mathcal{H}_2$ receive a lump sum income increase at this rate and this is weighted by the deviation of the marginal social utility of income of these households from the average. Since these households are in the upper income bracket, and σ_h is decreasing in utility v_h, we expect the sum of these deviations, weighted by the frequencies of the household types, to be negative. That is, the marginal cost of the bracket limit increase is a worsening in the equity of the income distribution. If however this right hand term were not positive, then this condition could not be satisfied and this would make untenable the assumption that $(\tau_2 - \tau_1) > 0$, in other words, that the optimal piecewise linear tax system is indeed convex.

4.2 Individual Taxation

The planner solves

$$\max_{a,t_1,t_2,y} \sum_{h=1}^{H} \phi_h S(v_h) \tag{39}$$

subject now to the public sector budget constraint

$$\sum_{\cup_{i=0}^{2} H_i} \phi_h t_1 y_h + \sum_{\cup_{i=3}^{4} H_i} \phi_h [t_1 y_{2h} + t_2 y_{1h} + (t_1 - t_2)\eta] + \sum_{H_5} \phi_h [t_2 y_h + 2(t_1 - t_2)\eta] \geq a \tag{40}$$

In what follows it will be useful to define $\delta_h \equiv (\sigma_h - 1)$, the deviation of the marginal social utility of income of a type h household from the mean, and $\mu_{ih} \equiv (1 - t_1) - \partial\psi/\partial y_{ih}$, the value of a relaxation of the bracket limit to an indivdual at the kink in the budget constraint. Then from the first order conditions for an optimal solution[23] we derive:

Proposition 2: The optimal tax parameters in the case of individual taxation are characterised by the following conditions.

$$\sum_{h=1}^{H} \phi_h \delta_h = 0 \tag{41}$$

$$t_1 = \frac{\sum_{H_0} \phi_h \delta_h y_{1h}^* + \sum_{H_0 \cup H_1 \cup H_3} \phi_h \delta_h y_{2h}^* + y[\sum_{H_2 \cup H_3 \cup H_4} \phi_h \delta_h] + 2y[\sum_{H_5} \phi_h \delta_h]}{\sum_{H_0} \phi_h \partial y_{1h}/\partial t_1 + \sum_{H_0 \cup H_1 \cup H_2} \phi_h \partial y_{2h}/\partial t_1} \tag{42}$$

$$t_2 = \frac{\sum_{H_3 \cup H_4 \cup H_5} \phi_h \delta_h (y_{1h}^* - y) + \sum_{H_5} \phi_h \delta_h (y_{2h}^* - y)}{\sum_{H_3 \cup H_4 \cup H_5} \phi_h \partial y_{1h}/\partial t_2 + \sum_{H_5} \phi_h \partial y_{2h}/\partial t_2} \tag{43}$$

$$\sum_{H_1 \cup H_2} \phi_h \sigma_h (\mu_{1h} + t_1) + \sum_{H_2 \cup H_4} \phi_h \sigma_h (\mu_{2h} + t_1) = -(t_2 - t_1)[\sum_{H_3 \cup H_4} \phi_h \delta_h + 2\sum_{H_5} \phi_h \delta_h] \tag{44}$$

4.3 Discussion

The key aspect of the change in tax systems is the finer partition of the set of household types which allows lower wage second earners to be taxed in the lower tax bracket. Given the stylised fact that second earners' labour suppliers are significantly more sensitive to net wage rate changes than those of primary earners, this is very likely to lead to a more progessive tax system, with the tax rate in the lower income bracket falling relative to that in the higher income bracket, a reduction in aggregate deadweight losses associated with the tax system, and a shift in the burden of taxation from households with relatively high

[23] Again, exactly which households will be in which subsets is determined at the optimum, and depends on the values of the tax parameters.

to households with relatively low second earner labour supplies. We support these qualitative conclusions here by comparing the conditions on the tax rates presented in Propositions 1 and 2. In Section 6 below we explore this further in a series of simulations.

By comparing the denominators of the expressions in (35), (36), (42), and (43), we see that as between the cases of joint and individual taxation, the denominators of the lower tax rate will increase and those of the higher tax rate will fall as a result of the switch of second earners to the lower tax bracket. This implies first, other things being equal, a fall in the lower bracket tax rate relative to that in the higher bracket, and so an increase in the progressivity of the tax system, and also a fall in overall deadweight loss.

In the numerators of the lower tax rate conditions in (35) and (42) there will be a decrease in the term representing the amount of lump sum tax revenue extracted from the upper tax bracket by the lower bracket tax rate, again as a result of the switch of lower wage second earners from the higher to the lower tax brackets, and this again tends to increase the progressivity of the tax system.

Finally, other things being equal we would expect an increase in the absolute values of the numerators of the expressions involving the upper bracket tax rates in (36) and (43). Note first that

$$y_{1h}^* - y > y_h^* - \eta \Leftrightarrow \eta - y > y_h^* - y_{1h}^* \tag{45}$$

We can interpret this as saying that the taxable portion of the primary earner's income in a higher wage household will be larger than the taxable portion of its joint income if the difference in the bracket limit on *joint income* and that on *individual income* is greater than the difference between joint income before the change in tax systems and primary income after it. If the primary earner's income hardly changes, this latter difference is approximately equal to the second earner's income under joint taxation. In a sense, this is a measure of the loss of tax advantage to a higher primary income household that arises from "income splitting". When this condition is satisfied, the first term in the numerator of (36) will, other things equal, be greater than the corresponding term in (43), and so again the higher bracket tax rate will tend to be relatively larger, and the tax system more progressive, under individual as opposed to joint taxation.

We should note of course that although these arguments help us to form an intuitive expectation of the qualitative effects of moving from joint to individual taxation in a two-bracket piecewise linear system, they do not provide a proof that these must occur. Other things will also change: the lump sum transfers; the bracket limits; the marginal social utilities of income; and the proportions of households in the respective subsets. For this reason the simulations presented in Section 6 below are also important.

5 Optimal tax analysis: Model 2

In applying this model to the optimal tax analysis, the key relationships are the indirect utility function and its derivatives with respect to the tax parameters.

The specifics of these will, as before, depend on whether we have individual or joint taxation. Indeed, we will show that on the face of it nothing changes in the expressions characterising the optimal tax parameters despite the fairly radical changes in the underlying household model we have just made. The essential reason for this is that when we reformulated the utility function in (6) in terms of earned income, we formulated a problem with the same budget constraint as in the case of Model 1, and so the derivatives of the indirect utility function, which are essentially determined by this constraint, take the same general form. What is important however is that because of the underlying model structure, both the interpretation of the optimal tax conditions and their policy implications change drastically.

5.1 Critique of joint taxation

The key point about joint taxation is that two households with the same income level but possibly widely different utility levels pay the same tax, and this is essentially due to the heterogeneity in second earner labour supply caused not by wage differences, but by differences in productivity in household production. We can use the above model to clarify this. The discussion is motivated by the following simple example. Suppose we observe two households, each earning a total household income of \$100,000. In household A this is earned entirely by the primary earner. In household B the primary earner contributes \$60,000 and the second earner \$40,000. Is it plausible that these households are equally well off in utility terms? Clearly, everything depends on the explanation of the heterogeneity in labour supply of the second earners in these households, their productivities in household production and the effect of this on total household output, both market and domestic.

To focus the discussion more sharply, assume perfect assortative matching, so that we can write $w_{2h} = \beta w_{1h}$, all $h = 1, 2, ..., H$. In the model just set out, given the solution values for the y_{ih} under the given tax system, in this case joint piecewise linear taxation, we can write total household income as a function

$$\sum_{i=1}^{2} y_{ih} = y_h = f(w_{1h}, k_h) \tag{46}$$

where we suppress the tax parameters since they are assumed constant throughout the following discussion. It is understood that variations in $k_{h,}$ and indeed w_{1h}, affect y_h essentially through variations in second earner income y_{2h}.

Consider now the set of households with (w_{1h}, k_h)-pairs satisfying the relationship

$$f(w_{1h}, k_h) = y_h^0 \tag{47}$$

where y_h^0 is a given optimal income level chosen by this set of households under the joint tax system. They are therefore all paying the same amount of tax. From the Implicit Function Theorem we have, given $\partial f/\partial k_h \neq 0$,

$$\frac{dk_h}{dw_{1h}} = -\frac{\partial f/\partial w_{1h}}{\partial f/\partial k_h} \gtreqless 0 \tag{48}$$

Since we have $\partial f/\partial w_{1h} > 0$, the sign of this expression depends on $\partial f/\partial k_h$. Thus we can distinguish two cases:

Case 1: Increasing productivity increases female labour supply and therefore household income, $\partial f/\partial k_h > 0$.

In this case we have $dk_h/dw_{1h} < 0$, so that, within the set of households with equal income, the lower the wage type, the higher must be its household productivity. To compensate for the effect of decreasing wage rates on household income, the income of the second earner must be increasing, and this arises in the case in which increasing household productivity increases second earner labour supply. (See Figure). In terms of our above example, household A must be at the bottom end of this curve and household B at the top end.

[Draw a figure with w_{1h} on the horizontal axis, k_h on the vertical and a downward sloping curve. B is on the top left hand part of the curve, A is on the bottom right hand part.].

Case 2: Increasing productivity reduces female labour supply and household income, $\partial f/\partial k_h < 0$.

In this case we have $dk_h/dw_{1h} > 0$, so that, within the set of households with equal income, the lower the wage type, the lower must be its household productivity. To compensate for the effect of increasing wage rates on household income, the income of the second earner must be decreasing, and this arises in the case in which increasing household productivity reduces second earner labour supply. (See Figure).[Draw a figure with w_{1h} on the horizontal axis, k_h on the vertical and an upward sloping curve.

[Draw a figure with w_{1h} on the horizontal axis, k_h on the vertical and an upward sloping curve. In terms of our above example, household A must be at the top right hand end of this curve and household B at the bottom left hand part.]

In terms of the comparative statics of the model, either of these cases is possible, and it is entirely an empirical question which of them holds.

In Case 1, there may not be a wide difference in household utility levels as we move along the curve, since the rising household productivity is at least to some extent compensating for the falling wage rates. However, households with the highest productivity and therefore highest second earner income may well be the least well off, because the higher productivity does not compensate, in terms of utility levels, for the lower wage rates.

Iin Case 2, clearly household utility possibilities vary inversely with household income, since both wage rates and productivity are falling as we move within the set of households with equal total income. In that case, a move to individual taxation leads to a welfare improvement, since it reduces the tax burden on second earners and shifts the tax burden to primary earners.

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