A Model of Voluntary Childlessness

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Abstract

Demographers and sociologists have studied and asked for a theory of childlessness for more than two decades, however, this specific choice of zero fertility has not interested economists. Nowadays, facts show us that permanent childlessness, in developed countries, can concern up to 30% of all women of a cohort. This paper provides an endogenous fertility model that looks in detail into the mechanisms leading to fluctuations in childlessness. Two mechanisms are considered. The first mechanism goes through the inter-generational evolution of preferences; I show that under certain parameter values, oscillatory dynamics of childlessness may arise. The second mechanism goes through the female labor market; a more gender parity labor environment and an increase in the fixed cost of becoming a parent could be an explanation for the dynamics of fertility and childlessness that we have observed in the United States since the beginning of the twentieth century.

Keywords: Fertility; Childlessness; Female Labor-Market Participation

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1 Introduction

Through the last three decades, developed countries have been facing a decrease in their fertility rates along with a general increase in the proportion of women remaining childless.[1] The economic literature on fertility in developed countries is large (Becker (1993), Galor and Weil (1996)) but to my knowledge, there is no work looking explicitly at the choice of remaining childless. A specific theory for childlessness is relevant because, counter to a priori expectation, fertility rates and childlessness are not always negatively correlated (see Section 2.1).

The increase in childlessness rates in developed countries has been explained by a combination of economic, social and cultural reasons, in line with the factors leading to the second demographic transition (van de Kaa (1987)). The aim of this work is to understand the mechanisms that can be responsible for the dynamics of voluntary childlessness. The literature about childlessness (Poston and Trent (1982), Morgan (1991), Toulemon (1996)) distinguishes between involuntary childlessness and voluntary childlessness. The first happens when the couple is unable to have children because of biological constraints leading to subfecundity.[2] Voluntary childlessness can either be defined in a restrictive way, such as couples who have never wanted children, or in a broader way, as couples who just happened to remain childless (Toulemon (1996)). I will use the last definition: voluntary childless couples are both who simply do not want to have children as well as who remain childless after a series of postponements (delaying childbearing is a more common attitude than a single decision to remain childless for life). This position can be discussed because postponements lead to a decrease in women's fecundity which may end up in an involuntary cause of childlessness. However, as economists who study rational individuals, it is natural for us to define these women as voluntary childless because women know from the beginning of their reproductive cycle that they are more fecund at age 25 than at age 35.

Poston and Trent (1982) are among the few who have proposed a theoretical analysis of childlessness. Considering the international variability of childlessness, they show that the difference in childlessness rates between countries is very large. Their main contribution is that there is a U-shaped relationship between childlessness and the development level of countries: childlessness in developing countries is high because a high proportion of women are affected by factors leading to subfecundity and consequently remain involuntarily childless, while childlessness in developed countries is high because women do not want to become mothers. As a country develops, childlessness decreases down to a minimum level and then increases because of voluntary reasons. The lowest childlessness rates correspond to

[1] In the United States, the *Census Bureau* reveals that the number of children ever born per woman aged 40 to 44 years old was 2.86 in 1981 and 1.86 in 2006; the childlessness rate for this cohort of women has increased from 9.5% to 20.4%.

[2] The causes leading to subfecundity are detailed in McFalls (1979)

an intermediate state of development. In this paper, I focus on developed countries since I am concerned with voluntary childlessness (see Figure 3). I do not look at the issue of involuntary childlessness because I consider that it has stabilized to its natural level[3] for developed economies and that today's fluctuations are due to the voluntary component.

In Houseknecht (1982), the author explains how voluntary childlessness is affected by three variables: female education, female labor employment and culture.[4] Here, I concentrate on the last two variables, and by culture I mean preferences over fertility. In the model, individuals have different tastes for children; this preference heterogeneity leads a fraction of the population to choose zero children. In a first part, I study whether the dynamics of preferences for children can explain the dynamics of childlessness, assuming that the labor of men and women are perfect substitutes. For this, I propose a simple model where the intergenerational transmission of preferences, from parents to children, is exogenous. An increase in gender parity, or a reduction in the gender wage gap, changing family compositions, allows for positive correlation between childlessness and fertility.

The model is calibrated to U.S. data. The purpose of this numerical exercise is to determine what do wage dynamics add to the simple dynamics of the first part. This is in line with the theoretical literature of fertility and female labor market, such as Galor and Weil (1996), where changes in relative wages of women with respect to men's can explain the dynamics of fertility rates, or Doepke et al. (2008), who look at how the change in labor demand during World War II influenced the Baby-Boom period.The simulations show that transitional dynamics to a steady state in the distribution of preferences over children are characterized by a negative correlation between fertility and childlessness rates, as one would expect. However, two experiments can, for some time, reserve this correlation. The first shows the impact of an increase in the productivity of women, similar to one of the experiments done in Jones et al. (2003), on fertility and childlessness. The second experiment looks at the impact of an increase in the fixed cost of becoming a parent on fertility and childlessness.

To my knowledge, there is no model that gives a complete analysis of the economic reasons leading a woman to remain childless; my contribution to the demographic economic literature is to provide a benchmark model that can account for the long run fluctuations of both fertility and childlessness. The first result is that a switch to a labor environment that gives more opportunities in the labor market to women can be a good explanation of the observed relationship between childlessness and

[3]For the Hutterites women who married before age 25, the completed childlessness rate is 2.4% (see Tietze (1957)).

[4]Lowly educated women associate children to social rewards and instruments to give meaning to their life, while for highly educated women, the social cost of remaining childless is covered by economic benefits and career commitments (see Blake (1979) and Houseknecht (1982)).

completed fertility for the cohorts born at the beginning of last century in the United States. The second result is that an increase in the cost of becoming parents during the mid XXth century can also reproduce empirical evidence on childlessness and fertility for cohorts born between 1930 and 1944, and in particular the positive relationship between both variables.

This paper is organized as follows. Section 2 provides an analysis of the existent literature on childlessness and analyzes childlessness over time and across countries. Section 3 presents the model. Section 4 studies the dynamics of childlessness assuming that men and women are perfect substitutes. Section 5 calibrates and simulates the model to U.S. data and Section 6 concludes.

2 Facts about childlessness

2.1 Childlessness and fertility

First of all, I would like to address the following question: is childlessness just a specific case of an endogenous fertility problem? Or, taking the question in an empirical perspective, is there a persistent link between completed fertility and childlessness? A first intuition would be that whenever fertility is high, childlessness is low and vice versa, in other words, we expect a negative correlation between fertility and childlessness. In the following paragraphs I look whether this negative correlation exists or not.

United States: In Figure 1, I plot both completed fertility, as children ever born (CEB), and childlessness for different cohorts of women born between 1840 and 1959 in the United States. A clear and unique negative relationship between both variables is not always present; the correlation coefficient is -0.52, which is quite low. So the statement that "as fertility declines, voluntary childlessness should increase"[5] is not that obvious.

Netherlands: For women born between 1900 and 1954 in the Netherlands, completed fertility has been decreasing and, except for the last cohort, childlessness has decreased as well (Figure 2). The correlation between both variables in this period of time is 0.60. This positive correlation shows that the choice of being childless is a different choice than the one of how many children a woman wants to have.

Cross country comparison: Another question to be asked is whether in countries with high fertility we find low childlessness and vice versa. Figure 3 gives the

[5] Poston and Trent (1982), page 477.

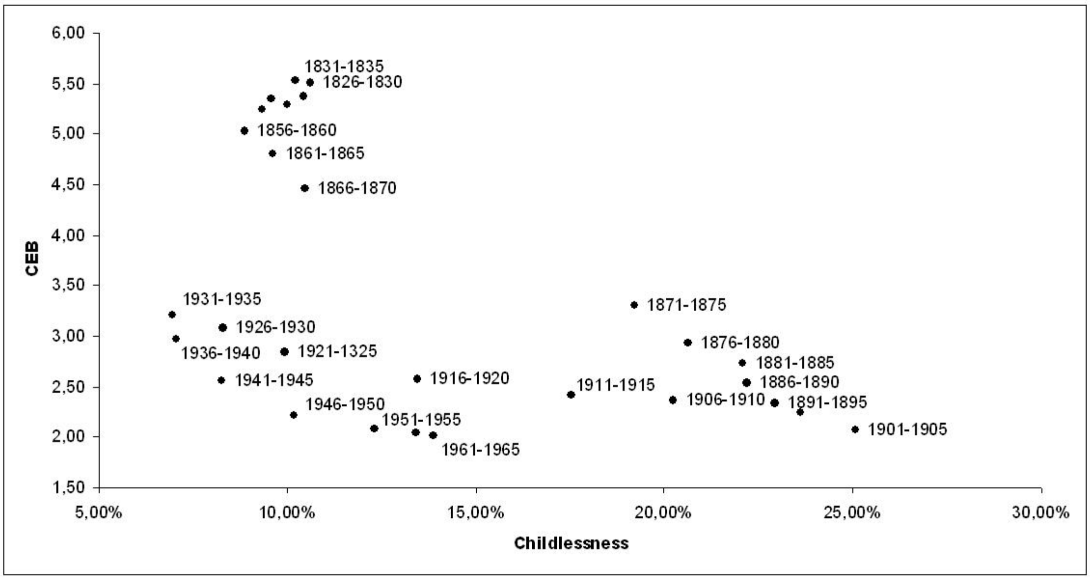

Figure 1: Relationship between childlessness and CEB for ever-married women born between 1826 and 1965 in the United States. See Table 1 of Appendix A and Figures 12 and 13 of Appendix B for details.

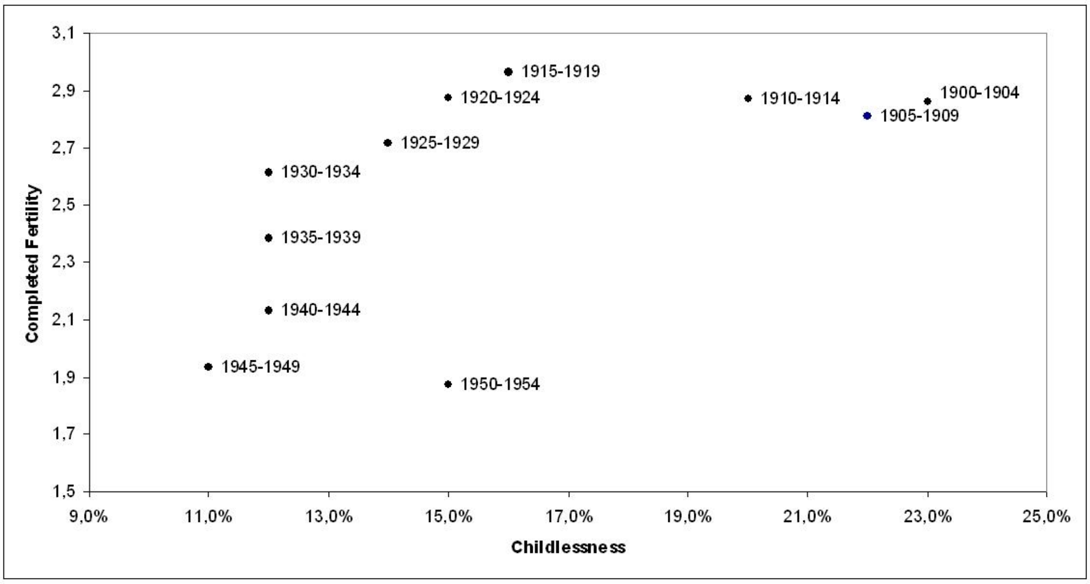

Figure 2: Relationship between childlessness and completed fertility for women born between 1900 and 1954 in Netherlands. See Table 2 of the Appendix for details.

relationship between both variables for some OECD countries for women born in 1965. The correlation between the two variables is -0.27 and it shows that a cross country analysis tells us, again, that fertility and childlessness do not have a clear negative relationship as we could have expected.

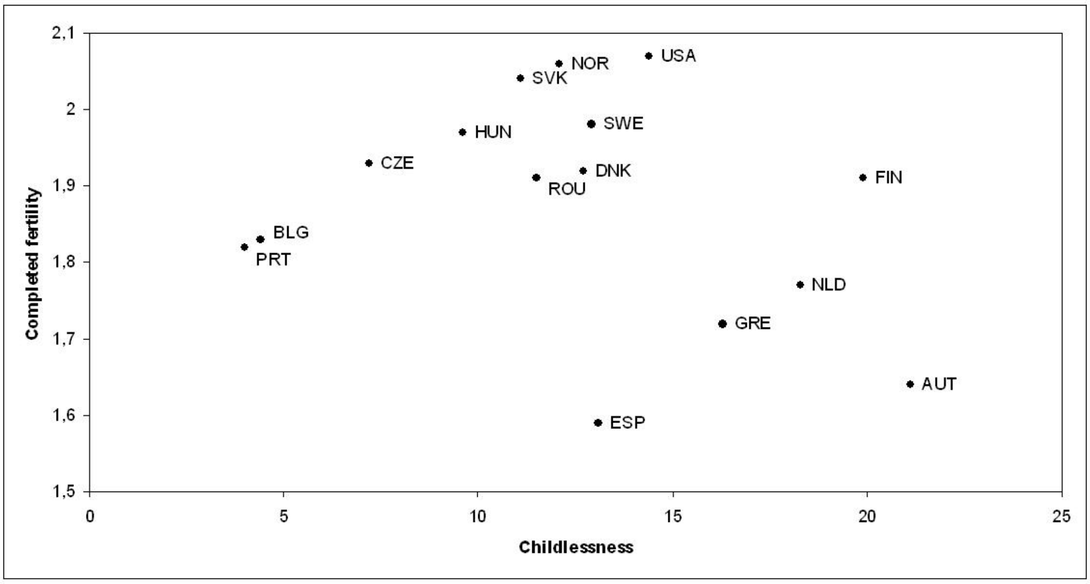

Figure 3: Relationship between childlessness and completed fertility for women born in 1965 in OECD countries. Note: See Table 3 of Appendix A for details.

The conclusion we can take from this brief exposition of facts is that fertility and childlessness are not correlated in a clear and unique way, both through time and across countries. This argument motivates and gives sense to this research that looks for a theory explaining voluntary childlessness, over and above fertility choice in general.

2.2 Childlessness over time

The evolution of the proportion of childless couples over time has been studied in detail by Merlo and Rowland (2000) for Australia, and in Rowland (2007), for other developed countries.

Both studies reveal three main features. First, a peak in childlessness rates for cohorts born between 1880 and 1910: around 20% to 30% of women remained childless. These women were in their reproductive age during the World Wars and the Great Depression. In Australia, the main factor of such a high childlessness rate was a rise in marital childlessness (childlessness among never married women actually decreases for these cohorts). Then, a pronounced decline in the proportions of childless women born between 1900 and 1940, until reaching minimum levels of 10%. The lowest percentages of childlessness happen for the cohorts that produced the Baby Boom (born between 1930 and 1940). This period was also exceptional because it was marked by unusual proportions of couples getting married and having children. And third, a revival of childlessness among more recent cohorts born after the Second World War. Predictions of the Australian Bureau of Statistics for women who are currently in their reproductive age claim that 28% will remain childless

Data tells us that childlessness rate fluctuations are very similar in developed countries. This fluctuations can be affected both by exogenous shocks, such as wars and depressions, and endogenous changes, such as the evolution of cultural and social norms (Noordhuizen et al. (2010)), as well as changes in the female labor market Juhn and Kim (1999). Figure 4 illustrates the evolution of childlessness among women born in different cohorts at ages 45-49 years old (it shows the evolution of childlessness among all women and not only married women).

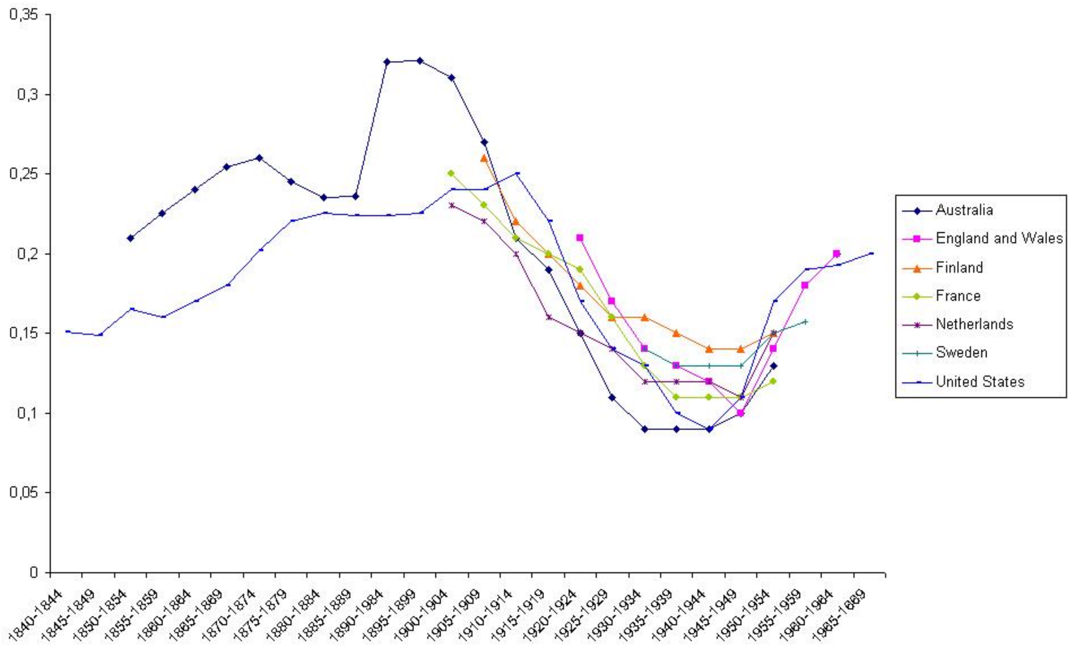

Figure 4: Evolution of childlessness for women born between 1840 and 1960. Sources: Rowland (2007), for the United States, cohorts 1950-1969 are taken from the US Census and for France, the last cohort is taken from Toulemon (1996). See and Figure 14 of Appendix B for the evolution of completed fertility for these countries.

3 The model

I consider an overlapping-generations model with an infinite discrete time framework in which individuals live for two periods: childhood and adulthood. Children consume a fixed amount of time from their parents, and adults take decisions about consumption and fertility. Fertility is measured in terms of couples of children, and a couple is made by a male and a female. During childhood, men and women are identical and when adults, they differ in their wages and the fact that only women have to give up some of their time to bear and raise children. Preferences over fertility can vary across individuals and couples.

3.1 Utility function of households

Members of a couple, j, take joint decisions about consumption, c_t^j, and the number (of couples) of children they want to have, n_t^j. The joint utility function of a couple j at time t is the following:

$$U_t^j\left(c_t^j, n_t^j\right) = \ln c_t^j + \gamma^j n_t^j \tag{1}$$

This utility function of the couple is an additively separable function of their consumption and the number of children they have. It is logarithmic in the couple's consumption, in line with Barro and Becker (1986), where the utility of a parent from consumption is given by an increasing and concave utility function. The couple's utility also depends linearly on the number of children they have, this is similar to the framework used in Barro and Becker (1986) except that in their case, the utility of children is incorporated into the utility of parents, while I suppose that parents are not altruistic in their children's utility, but rather in their lives[6]. The variable $\gamma^j > 0$ multiplying the fertility decision of the couple, represents couple j's willingness of having children. γ^j is given by an average between the woman and the man's taste for children of a couple (each member of the couple has the same bargaining power).

3.2 Constraint

Each adult has one unit of time; men use this time for working and women use it either for working or to bear and raise the couple's children. The assumption that only women raise children is realistic: there is a biological fact that men do not get pregnant or breastfeed but there is also a social component revealed by the fact that among parents of children under 18 who are full-time workers, married mothers are more likely to provide childcare to the children and to do household activities than fathers[7]. Raising children implies an opportunity cost for women:

[6] See Appendix D for a logarithmic utility function in n_t^j: $U_t^j(c_t^j, n_t^j) = \ln\left(c_t^j\right) + \gamma^j \ln\left(\mu + n_t^j\right)$ where μ is another preference parameter giving the utility of being childless.

[7] *U.S. Bureau of Labor Statistics* in the release *Married Parents' Use of Time Summary* (2008).

the time spent with children is no longer available to work and the higher her wage, the higher this opportunity cost.

Parents face both a time cost and a fixed cost. The time cost, $\theta \in [0,1]$, is related to the bearing and raising of a child; it includes the pregnancy and breast-feeding time as well as the home production tasks such as cleaning, cooking and transport. The fixed cost, k, can be interpreted as a start-up cost to having children, this can be buying a larger house, buying a car, preparing the first pregnancy or life insurance. It can also be seen as an obligation to protect and raise children[8]. This fixed cost is also present in Bick (2010), who points out the fact that k could also be negative, meaning that a couple receives an utility gain for the births of its first child. Empirical evidence for the presence of this type of cost is given in Espenshade (1977), who shows the difference in terms of costs of a first child compared to the second. Accordingly, the household constraint is the following one,

$$c_t^j = w_t^m + \left(1 - \theta n_t^j\right) w_t^f - kI\left(n_t^j\right) \tag{2}$$

where w_t^m and w_t^f are respectively the wages per unit of time for men and for women at time t. The dichotomic variable, $I(n_t^j)$, differentiates the constraint between childless and non-childless couples in the following way,

$$I(n_t^j) = \begin{cases} 0 & \text{if} \quad n_t^j = 0 \\ 1 & \text{if} \quad n_t^j > 0 \end{cases}$$

3.3 The household problem

A couple j solves the following problem,

$$\max_{c_t^j, n_t^j} \quad U_t^j(c_t^j, n_t^j) = \ln(c_t^j) + \gamma^j n_t^j$$

$$s.t. \qquad c_t^j = w_t^m + (1 - \theta n_t^j) w_t^f - kI(n_t^j)$$
$$\text{and} \qquad 0 \leq n_t^j \leq \frac{1}{\theta}$$

where

$$I(n_t^j) = \begin{cases} 0 & \text{if} \quad n_t^j = 0 \\ 1 & \text{if} \quad n_t^j > 0 \end{cases}$$

[8]Quoting Dasgupta (2005):

> "People do not have an obligation to become parents, of course, but they acquire one toward their children if they choose to become parents."

This is in line with what is stated in the United Nations Convention on the Rights of the Child text. A last interpretation, and close to the precedent, is the fact that the loss of freedom and flexibility of a couple are mainly related to the coming of the first child (Espenshade (1977)).

There are three possible solutions to this problem: two corner solutions, $(n^{n_{\max}t}, c_t^{n_{\max}})$ and (n_t^0, c_t^0), given by,

$$\begin{cases} n_t^{n_{\max}} = \frac{1}{\theta} \\ c_t^{n_{\max}} = w_t^m - k \end{cases}$$

and

$$\begin{cases} n_t^0 = 0 \\ c_t^0 = w_t^m + w_t^f \end{cases}$$

and the following interior solution (n_t^*, c_t^*),

$$\begin{cases} n_t^* = \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{1}{\gamma^j} \\ c_t^* = \frac{\theta w_t^f}{\gamma^j} \end{cases}$$

Proposition 3.1. *There exists a unique value of* $\gamma^j \equiv \gamma^*$ *for which couples are indifferent between being childless or not.*

Proof. See Appendix C. □

This allows us to define two types of couples,

1. The ones with high willingness for children, with $\gamma^j \geq \gamma^*$, who become parents.

2. The ones with low willingness for children, with $\gamma^j < \gamma^*$, who remain childless.[9]

As shown in Appendix C, an increase in the wage of men (or a decrease in the fixed cost) decreases the critical level γ^*.

Note that with no fixed cost $(k = 0)$, we would still have childlessness for values such that $\gamma^j \leq \frac{\theta w_t^f}{w_t^m + w_t^f}$.

3.4 Three types of marriages

I assume that there are only two values for the individual's taste for children: a low value, $\underline{\gamma}$, for the individuals with low taste for children and a high value, $\overline{\gamma}$, for the individuals with high taste for children. Since both members of a couple have the same bargaining power in the decision to have children, there are three different types of couples:

1. $(\underline{\gamma}\ \underline{\gamma})$: couple $j = 1$, characterized by $\gamma^1 = \underline{\gamma} < \gamma^*$ so that it remains childless

2. $(\overline{\gamma}\ \underline{\gamma})$: couple $j = 2$, with $\gamma^2 = \frac{\overline{\gamma} + \underline{\gamma}}{2} \geq \gamma^*$ and a fertility rate $\overline{n}$

[9] These are often called DINKS in the media or the marketing literature, standing for "double income, no kids".

3. $(\overline{\gamma}\ \overline{\gamma})$: couple $j = 3$, with $\gamma^3 = \overline{\gamma} > \gamma^2$ and the highest fertility rate $\overline{\overline{n}}$.

Accordingly, the fertility of couples $j = 2$ and $j = 3$ will be the following,

$$\overline{n}_t = \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{2}{\overline{\gamma} + \underline{\gamma}}$$

$$\overline{\overline{n}}_t = \frac{w_t^m + w_t^f - k}{\theta w_t^f} - \frac{1}{\overline{\gamma}}$$

At time t, total population, P_t, is given by by the sum of individuals with high taste for children, $\overline{P_t}$, and individuals with low taste for children, $\underline{P_t}$:

$$P_t = \underline{P_t} + \overline{P_t} \tag{3}$$

Random matching: I assume that couples match randomly.[10] This allows to compute the proportions of each type of marriage at time t:

1. The proportion of marriages of type 1 is: $\left(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2$
2. The proportion of marriages of type 2 is: $\frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}$
3. The proportion of marriages of type 3 is: $\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2$

At time t, average fertility, n_t is given by,

$$n_t = \left(\frac{\overline{P_t}}{\overline{P_t} + \underline{P_t}}\right)^2 \overline{\overline{n_t}} + \frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t} + \underline{P_t}\right)^2}\overline{n_t} + \left(\frac{\underline{P_t}}{\overline{P_t} + \underline{P_t}}\right)^2 0$$

that can be simplified as,

$$n_t = \frac{\overline{P_t}}{(\overline{P_t} + \underline{P_t})^2}\left[\overline{P_t}\,\overline{\overline{n_t}} + 2\underline{P_t}\,\overline{n_t}\right] \tag{4}$$

3.5 Production function

A representative competitive firm, producing the final good, Y_t, used for consumption at unit price, and using men's labor, L_t^m, and women's labor, L_t^f (both in units of time), as inputs, has the following CES production function,

$$F(L_t^m, L_t^f) = Y_t = \left(\alpha(L_t^m)^{-\rho} + (1-\alpha)(L_t^f)^{-\rho}\right)^{-1/\rho} \tag{5}$$

[10] In Appendix E, I discuss this hypothesis and show that an assortative matching framework does not change the results.

with $\alpha \in (0,1)$ and $\rho \geq -1$, $\rho \neq 0$. Women's labor, L_t^f is equal to the sum of the labor of childless women, the labor of women having few children, $\overline{n_t}$, and the labor of women having many children, $\overline{\overline{n_t}}$. The amount of time that a woman spends working will then be,

$$\begin{cases} 1 & \text{if childless} \\ 1-\theta\overline{n_t} & \text{if she has } \overline{n_t} \text{ (couples of) children} \\ 1-\theta\overline{\overline{n_t}} & \text{if she has } \overline{\overline{n_t}} \text{ (couples of) children} \end{cases}$$

We will denote by L^{f1}, L^{f2} and L^{f3} the labor supplied by childless women, women with $\overline{n}$ children and women with $\overline{\overline{n}}$ children respectively. Total female labor supply, given by the total number of hours worked by all women, at time t, is then given by,

$$L_t^f = L_t^{f1} + L_t^{f2} + L_t^{f3}$$

The representative firm solves the following problem,

$$\max_{L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}} \quad F\left(L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}\right) - w_t^m L_t^m - w_t^f\left(L_t^{f1} + L_t^{f2} + L_t^{f3}\right)$$

Equalizing the marginal productivities of labor to their marginal cost, we find,

$$w_t^m = \alpha\left(\alpha + (1-\alpha)\left(\frac{L_t^f}{L_t^m}\right)^{-\rho}\right)^{-\frac{1+\rho}{\rho}}$$

$$w_t^f = (1-\alpha)\left(\alpha\left(\frac{L_t^m}{L_t^f}\right)^{-\rho} + (1-\alpha)\right)^{-\frac{1+\rho}{\rho}}$$

So, the wage of men increases as the female labor supply increases and decreases as men's labor increases (except for $\rho = -1$ where wages do not depend on the amount of labor). The same happens for the wage of women; an increase in female labor supply decreases women's wage and an increase in male's labor supply increases it.

At time t, total population, P_t, given by Equation (3), is composed by one half of men, P_t^m, and the other half of women, P_t^f. Total labor supplies for each type of person are then the following ones,

$$L_t^m = \frac{\underline{P_t} + \overline{P_t}}{2}$$

$$L_t^{f1} = \frac{\underline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}$$

$$L_t^{f2} = \frac{\overline{P_t}\underline{P_t}}{\underline{P_t} + \overline{P_t}} \left(1 - \theta \overline{n_t}\right)$$

$$L_t^{f3} = \frac{\overline{P_t}^2}{2(\underline{P_t} + \overline{P_t})} \left(1 - \theta \overline{\overline{n_t}}\right)$$

The total amount of time supplied by women in terms of population groups is then given by,

$$L_t^f = \frac{\underline{P_t}^2}{2(\underline{P_t} + \overline{P_t})} + \left(1 - \theta \overline{n_t}\right) \frac{\overline{P_t}\underline{P_t}}{\underline{P_t} + \overline{P_t}} + \left(1 - \theta \overline{\overline{n_t}}\right) \frac{\overline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}$$

Definition 3.1 (Temporary Equilibrium:)**.** *Given adult population groups* $(\underline{P_t}, \overline{P_t})$ *characterized by their respective willingness for children* $(\underline{\gamma}, \overline{\gamma})$*, a temporary equilibrium is a vector*

$$\{c_t^j, n_t^j, \gamma^j, z_t, , P_t^m, P_t^f, P_t, L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}, Y_t, w_t^m, w_t^f\}$$

satisfying the following conditions:

- *the level of the couple's consumption,* c_t^j*, and the fertility of the couple,* n_t^j*, is such that each couple* j *maximizes its utility* $U_t^j(c_t^j, n_t^j) = \ln c_t^j + \gamma^j n_t^j$ *subject to the constraints* $c_t^j = w_t^m + (1 - \theta n) w_t^f - kI(n_t^j)$ *and* $0 \leq n_t^j \leq \frac{1}{\theta}$*;*
- *couples match randomly and the willingness for children of the couple* γ^j *is given by an average of the tastes of its members so that there are three types of couples characterized by different willingnesses:* $\underline{\gamma}$*,* $\frac{\underline{\gamma}+\overline{\gamma}}{2}$ *and* $\overline{\gamma}$*;*
- *the relative size of population, at time* t*,* z_t*, is given by* $z_t = \frac{\overline{P_t}}{\underline{P_t}}$*;*
- *total population,* P_t*, at time* t*, has an equal number of men and women:* $P_t^m = P_t^f = \frac{P_t}{2}$ *and is given by* $P_t = \underline{P_t} + \overline{P_t}$*;*
- *labor inputs* $L_t^m, L_t^{f1}, L_t^{f2}$ *and* L_t^{f3} *and output level* Y_t *are such that the competitive firm maximizes its profits given by:* $Y_t - w_t^m L_t^m - w_t^f (L_t^{f1} + L_t^{f2} + L_t^{f3})$*, and produces,*

$$Y_t = \left(\alpha \left(L_t^m\right)^{-\rho} + (1 - \alpha)\left(L_t^{f1} + L_t^{f2} + L_t^{f3}\right)^{-\rho}\right)^{-1/\rho}$$

- *wages per unit of time,* w_t^m *and* w_t^f*, are such that the labor market clears:*

$$L_t^m = \frac{\underline{P_t} + \overline{P_t}}{2}$$

$$L_t^{f1} = \frac{\underline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}$$

$$L_t^{f2} = \frac{\underline{P_t}\,\overline{P_t}}{\overline{P_t} + \underline{P_t}}\,(1 - \theta\overline{n_t})$$

$$L_t^{f3} = \frac{\overline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}\left(1 - \theta\overline{\overline{n_t}}\right)$$

3.6 Comparative statics

Effect of a change in w_t^m on fertility: From the fertility of the couple given by the interior solution of the couple's maximization problem, we can check that holding women's wage constant, fertility is increasing with men's wage. The reason is due to the assumption that all childrearing is done by women, therefore, an increase in men's wage has a pure income effect on the fertility decision of a couple.

Effect of a change in w_t^f on fertility: Keeping men's wage constant, an increase in women's wage has both an income effect and a substitution effect on fertility: it raises the overall income of the couple but the time that is not dedicated to work becomes more expensive.

$$\frac{\delta n_t^{j*}}{\delta w_t^f} = \frac{k - w_t^m}{\theta (w_t^f)^2}$$

Fertility can then be either increasing or decreasing with women's wage:

- $k > w_t^m \Rightarrow \frac{\delta n_t^j}{\delta w_t^f} > 0$; in order to have $n_t^j > 0$ for this case, then $\gamma^j > \theta$ must hold because,

  $$\lim_{w_t^f \to \infty} n_t^j = \frac{1}{\theta} - \frac{1}{\gamma^j}$$

  This can be interpreted in the following way: when the willingness for children of a couple is higher than the time cost of raising children, and men's wage is not high enough to cover the fixed cost of having children, the income effect of an increase in the woman's wage will dominate the substitution effect, and fertility will increase. This positive relation between women wages and fertility, due to the presence of a fixed cost of having children, is a particularity of this model and it could partly explain the relatively higher fertility levels of lower income groups. When the time cost of the children is larger than the willingness for children, the couple will have no children for any increase in the woman's wage.

- $k < w_t^m \Rightarrow \frac{\delta n_t^j}{\delta w_t^f} < 0$; if the fixed cost of having children is covered by men's wage (which is most likely to occur), we will have that a higher w^f will reduce the fertility of the couple, since the higher wage increases the opportunity cost of having children more than the household income. This negative relationship between women wages and fertility is what we usually find in the literature (Galor and Weil (1996)). Without the fix cost k, and keeping the same structure of the model, this inverse relationship between fertility and women's wage would always hold.

The following figure illustrates the relationship between the wage of women and fertility in the case where $k < w^m$ and $\gamma^j < \theta$. We can see that fertility will be constant and equal to the corner solutions for either a very low wage for women ($n^j = n^{n_{\max}}$) or a high wage for women ($n^j = n^0$). At the interior solution, a higher female wage decreases fertility.

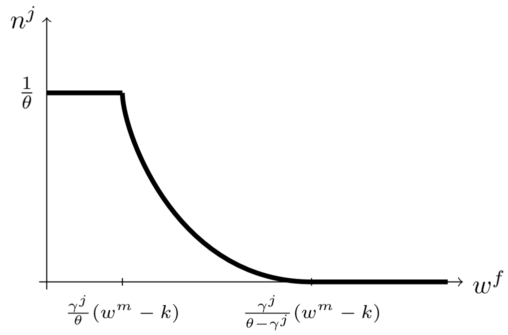

Figure 5: Couple's fertility as a function of women's wage

4 Dynamics

In this section I look at the dynamics of population groups. The main question here is whether a model of inter-generational transmission of preferences can explain the dynamics of childlessness. I assume that exogenous probabilities relate the willingness for children of a couple to the taste for children that a child of this couple will have in the next period. An analysis with endogenous probabilities is computed in Appendix G.

To make the model treatable analytically, I assume in this part that male and female labor are perfectly substitutable ($\rho = -1$). This implies that wages will be equal to the weight of each input in the production of the final good:

$$w^m = \alpha$$

$$w^f = 1 - \alpha$$

Let a be the probability of having a child with $\overline{\gamma}$ in a marriage of type $(\overline{\gamma}\,\overline{\gamma})$, and b the probability of having a child with $\overline{\gamma}$ in a marriage of type $(\overline{\gamma}\,\underline{\gamma})$. I will treat here the case where $1 > a > b$ (the other two cases, $a = b$ and $a < b$, are treated in Appendix F).

The dynamics for the two groups of individuals are given by the following equations,

$$\overline{P_{t+1}} = 2a\overline{\overline{n}} \underbrace{\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \frac{\overline{P_t}+\underline{P_t}}{2}}_{\text{number of marriages of type } \overline{\gamma}\overline{\gamma}} + 2b\overline{n} \underbrace{\frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}\frac{\overline{P_t}+\underline{P_t}}{2}}_{\text{number of marriages of type } \overline{\gamma}\underline{\gamma}}$$

and

$$\underline{P_{t+1}} = 2(1-a)\overline{\overline{n}}\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \frac{\overline{P_t}+\underline{P_t}}{2} + 2(1-b)\overline{n}\frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}\frac{\overline{P_t}+\underline{P_t}}{2}$$

Definition 4.1 (Intertemporal equilibrium:)**.** *Given initial young population groups* $(\underline{P_0}, \overline{P_0})$*, an intertemporal equilibrium is a sequence of temporary equilibria such that population groups follow the following expressions,*

$$\overline{P_{t+1}} = \frac{1}{\overline{P_t}+\underline{P_t}}\left(a\overline{\overline{n}}\,\overline{P_t}^2 + 2b\overline{n}\,\overline{P_t}\underline{P_t}\right) \tag{6}$$

and

$$\underline{P_{t+1}} = \frac{1}{\overline{P_t}+\underline{P_t}}\left((1-a)\overline{\overline{n}}\,\overline{P_t}^2 + 2(1-b)\overline{n}\,\overline{P_t}\underline{P_t}\right) \tag{7}$$

These two equations, describing the dynamics of the groups, can also be expressed by a single difference equation of order one;

$$z_{t+1} = \frac{a\overline{\overline{n}}z_t + 2b\overline{n}}{(1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}} \equiv \phi(z_t) \tag{8}$$

where $z_t = \frac{\overline{P_t}}{\underline{P_t}}$ is the relative group of individuals with high taste for children. Computing the first and second order derivative of $\phi(z_t)$ we have that,

$$\phi'(z_t) = \frac{2\overline{\overline{n}}(a-b)\overline{n}}{\left((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\right)^2} > 0$$

and

$$\phi''(z_t) = \frac{-4\overline{\overline{n}}^2\overline{n}(a-b)(1-a)}{\left((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\right)^3} < 0$$

and we can easily see that,

$$\phi(0) = \frac{b}{1-b} > 0$$

$$\lim_{z_t \to \infty} \phi(z_t) = \frac{a}{1-a} > 0$$

The proportion of childless women at time t, denoted by χ_t, can be expressed in terms of z_t as follows,

$$\chi_t = \frac{1}{(z_t+1)^2} \tag{9}$$

and the average fertility, n_t can also be rewritten as,

$$n_t = \frac{z_t}{(1+z_t)^2} \left(z_t \overline{\overline{n}} + 2\overline{n} \right) \tag{10}$$

Definition 4.2 (Steady State)**.** *We define a steady state, a state where the relative group of individuals with high taste for children,* $z_t = \frac{\overline{P_t}}{\underline{P_t}}$*, is constant over time, so that* $z^* = z_t = z_{t+1} = \ldots$*.*

From Equation (8), we see that there is a unique, positive, steady state, z^*, where $z^* = \phi(z^*)$, equal to;

$$z^* = \frac{-\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right) + \sqrt{\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right)^2 + 2(1-a)\overline{n}b\overline{\overline{n}}}}{(1-a)\overline{\overline{n}}}$$

The function $\phi(z_t)$ is strictly increasing and concave in $\mathbb{R}_+$, consequently, the dynamics of z are monotonic and converge to z^* whatever the initial condition z_0. Starting at a time $t = 0$, from any level $z_0 < z^*$, we have $z_t < z_{t+1}$, meaning that the group of individuals with high taste for children increases relative to the group of people who dislike children until reaching the steady state level z^*. Reversely, from an initial value $z_0 > z^*$, we have $z_t > z_{t+1}$ and consequently the dynamics are decreasing. Since from any initial level z_0 we converge to the steady state level z^*, we can say that this steady state is globally stable in $\mathbb{R}_+$. Figure 6 illustrates this.

The steady state value, z^*, is strictly positive, this means that, in the long run, none of the two groups will become extinct. If $z^* = 0$, this would imply that the population with high taste taste for children would disappear and if we had $z^* = \infty$, then the population disliking children would disappear. None of these cases are possible here, so both groups will always be present.

Proposition 4.1. *If* $a > b$*, the dynamics of* z_t *are monotonic, and converge to a unique globally stable steady state.*

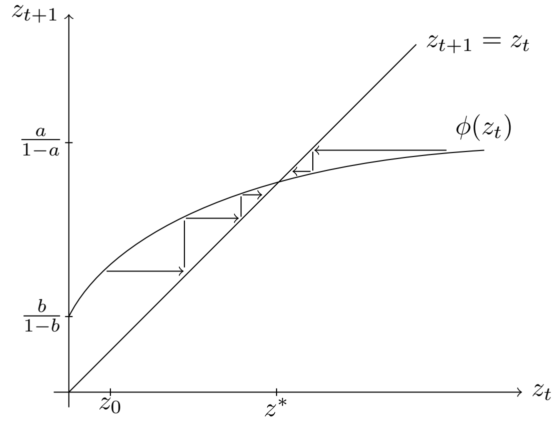

Figure 6: Monotonic dynamics in the case $a > b$

The intuition behind the dynamics becomes clear if we consider respectively the proportion of children coming from marriages of type 2 (mixed marriages) and of type 3:

$$\frac{2\overline{n}}{\overline{\overline{n}}z_t + 2\overline{n}} \tag{11}$$

and

$$\frac{\overline{\overline{n}}z_t}{\overline{\overline{n}}z_t + 2\overline{n}} \tag{12}$$

Considering a case where z_0 is initially low ($z_0 < z^*$), the proportion of children coming from marriages of type 2 is high compared to children coming from marriages of type 3. Since $a > b$, the proportion of children coming from marriages of type 3 will increase. This increases z until it reaches the steady state.

Monotonic dynamics are not present in the reality described in the first part of the paper. However, it still brings a positive result from numerical simulations: a negative shock on α, meaning a higher weight for women's labor inside the firm, that increases w^f (and decreases w^m) increases z^* and decreases both n^* and χ^*. This is interesting because this very simple model allows for a positive correlation between childlessness and fertility. The intuition is that a decrease in α decreases $\overline{n}$ and $\overline{\overline{n}}$ by the same amount, however, for couples of type 2, this decreases is bigger in relation of the number of children they had before the shock than for couples of type 3, and since the fertility of the couples that are less likely to have children with high taste for children is the most affected one, then z^* increases, and n^* and χ^* decrease. This case is represented in Figure 8 for a value of $\rho = -0.75$ and calibrated parameters.

A remark on the timing of the changes in fertility and childlessness can also be helpful for understanding the intuition of the mechanism. A decrease in α has an immediate impact on fertility rates of parents. However, the effect on childlessness will be seen a period after, since a lower proportion of children of type 2 implies that, when this generation arrives to adulthood, their children will be less likely to be childless. Consequently, the effect of this kind of shock affects first fertility rates and then childlessness.

We can conclude that this simple model can explain a positive relationship between childlessness and fertility rates. For an analysis when $a \leq b$, see Appendix F.

5 Calibration and simulations for the United States

In this section, I consider the full blown model and study what wage dynamics add to the simple dynamics of Section 4. I study the effects on childlessness, fertility, female labor market participation and wage gap between men and women of a change in two parameters: the weight of women in the production of the final good and the fixed cost of going from childlessness to parenthood. For this, I fix two parameters and calibrate the rest of them in order to match United States data.

5.1 Calibration

The following two parameters are a priori fixed: b and ρ. I set the probability $b = 0.8a$; this is arbitrary but the only value that is affected by changing this restriction is the probability a, that increases if the ratio b/a decreases. The other variables remain unchanged; however, the dynamics are slower when the ratio b/a is small. The substitution parameter is also a priori fixed to $\rho = -0.75$, implying an elasticity of substitution between female labor and male labor of 4. This choice is coherent with the estimates of Acemoglu et al. (2004). Changing ρ affects the distribution parameter α: the lower the substitution between the inputs, the higher will be the weight of men inside the firm.

Equations used for the calibration: At the steady state, we have a system of the following eight equations (with eight unknowns[11]),

$$z = \frac{a\overline{\overline{n}}z + 2b\overline{n}}{(1-a)\overline{\overline{n}}z + 2(1-b)\overline{n}}$$

$$\begin{cases} w^m = \alpha \left(\alpha + (1-\alpha) l^{-\rho}\right)^{-\frac{1+\rho}{\rho}} \\ w^f = (1-\alpha)\left(\alpha l^{\rho} + (1-\alpha)\right)^{-\frac{1+\rho}{\rho}} \end{cases}$$

[11]The unknowns are θ, $\underline{\gamma}$, $\overline{\gamma}$, a, z, α, w^m and $\overline{n}$.

$$\begin{cases} \overline{n} = \frac{w^m + w^f - k}{\theta w^f} - \frac{2}{\overline{\gamma} + \underline{\gamma}} \\ \overline{\overline{n}} = \frac{w^m + w^f - k}{\theta w^f} - \frac{1}{\overline{\gamma}} \\ n = \frac{z}{(1+z)^2} \left(z\overline{\overline{n}} + 2\overline{n} \right) \end{cases}$$

$$l = \frac{1 + 2\left(1 - \theta\overline{n}\right) z + \left(1 - \theta\overline{\overline{n}}\right) z^2}{\left(1 + z\right)^2}$$

and

$$\chi = \frac{1}{(1+z)^2}$$

where $l = \frac{L^f}{L^m}$ is the relative labor supplied by women with respect to men.

I use the study of Turchi (1975)[12] to calibrate the fixed cost, k, of going from childless to parents. Considering that childrearing is done for 18 years, two children cost 12946 hours while four children cost 23832 (assuming that the fourth kid costs you the same than the third). This means that the first two cost 2060 hours more. A period of life is 25 years, so we have the following restriction for the fixed cost:

$$k = 0.0047 \frac{w^m + w^{f2}}{2}$$

The other five parameters; θ, α, $\overline{\gamma}$, $\underline{\gamma}$ and a, are set to match five moments, taken from US data; n, χ, $\overline{\overline{n}}$, w^f and l. The following table gives us the value of the moments and the calibrated parameters:

Parameters	Moments	Source
$a = 0.670$	$n = 1$	
$\underline{\gamma} = 0.141$	$\chi = 0.146$	U.S. Census Bureau 2008
$\overline{\gamma} = 0.191$	$\overline{\overline{n}} = 1.97\overline{n}$	U.S. Census Bureau 2008
$\theta = 0.333$	$l = 0.667$	Erosa et al. (2005)
$\alpha = 0.587$	$w^f = 0.78w^m$	Erosa et al. (2005)

The rest of the variables take the following values:

$z = 1.617$	$w^m = 0.565$	$\overline{n} = 0.817$	$k = 0.00236$	$\gamma^* = 0.157$

Notes:

- Relative labor supply l: quoting Erosa et al. (2005), "We document that the average number of hours of work per person is about 40% larger for men than for women between the ages of 20 and 40. By age 40, this difference in hours of work translates into a stock of accumulated experience that is about 50% larger for men than for women." (page 3), this implies that $l = 0.667$.

[12]Turchi (1975), Table 3-5, page 92.

- Opportunity cost θ: A value of $\theta = 0.333$ implies a maximum fertility of around 6 children for a woman. According to Livi-Bacci (1977), considering the Hutterites' hypothetical number of children per woman, this is 8.2 if the women gets married at 25 years old[13]. Our calibration for θ may then be higher than the one expected but, if we look at the data, only 0.5% of all women have seven or more children in the United States[14].

5.2 Simulation

Using the parameters calibrated in the last subsection, the dynamics of the relative population z_t are monotonic and the correlation between average fertility and childlessness along the transition path is negative. This means that if we consider an initial condition, $z_0 = 3$, with a high proportion of individuals with high taste for children, the dynamics of z will be decreasing and we will see the following relationship between childlessness and fertility:

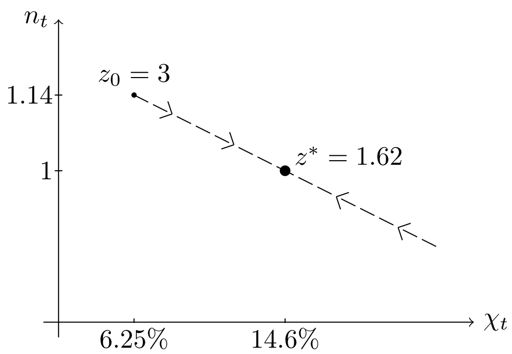

Figure 7: Correlation between n_t and χ_t along the transition path.

Starting with a high proportion of individuals with high taste for children, the proportion of children coming from the mixed type of couples is low compared to the proportion of children coming from type 3 households. Since probabilities are such that $a > b$, the proportion of mixed couples in the next period increases, which increases childlessness. In the labor market, the relationship between childlessness and the wage gap long the balanced growth path is positive: more full time working women increases the labor supplied by women, decreasing women's wage and increasing men's wage. Along the balanced growth path, the relationship between childlessness and relative labor supplied by women is non monotonic: higher childlessness increases l, but since w^f decreases and w^m increases, fertility of mothers increases, which has a negative impact on l. The overall effect on average fertility

[13]Livi-Bacci (1977), Table 1.2.
[14]U.S. Census Bureau for 40-44 years old women in 2006.

is negative due to the increase in childlessness and the increase in the proportion of mixed couples.

Note: If we calibrate and then simulate with $\rho = 1$, instead of $\rho = -0.75$, meaning that L^m and L^f are complements in the production of the final good, then the parameter that changes the most is the distribution parameter α, which increases to 0.742. The dynamics of z remain monotonic and the relationships between n and χ, l and χ and w^f/w^m and χ, along the transition path remain the same as when L^m and L^f are substitutes.

5.3 Simulation with shocks

Here I compute two experiments. The first is similar to Jones et al. (2003) who look at the effects on the labor supplied by married women of a decrease in wage discrimination. The second studies the impact of an increase in the fixed cost of parenthood.

More gender parity (decrease in α): A negative shock on the distribution parameter α of the production function means that female labor has a bigger weight in the production of the final good of the representative firm. Empirical evidence for this type of shock is supported by the results of O'Neill and Polachek (1993). Supposing that the economy is at the steady state before the shock, and that α has decreased of 10%, the initial condition is $z^*_{\alpha=0.65} = 1.55$.

Figure 8 shows how the shock affects average fertility and childlessness: both variables decrease after the shock. The intuition behind is the following: the shock

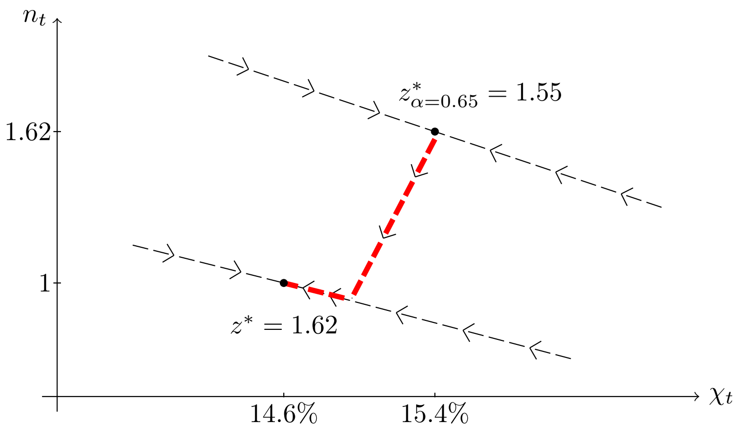

Figure 8: Effect of a decrease in α on n_t and χ_t.

increases the wage of women and this decreases the fertility of mothers, $\overline{n}$ and $\overline{\overline{n}}$; at the same time, it decreases the proportion of children coming from couples of type 2, which are the most likely to end up being childless (the proportion of children coming from couples of type 3 increases). In other words, small families shrink more than big families and since the latter are less likely to become childless, we have that childlessness decreases. In Figure 8, we see that average fertility decreases and then increases a little, the reason for the last increase is due to the rise in the proportion of women having the highest fertility. Coming back to the United States' relationship between childlessness and fertility (Figure 1), this could be an explanation of what happened for the cohorts born at the beginning of the nineteenth century up to the cohorts born before 1935.

In the labor market, the shock has a negative impact on men's wage and a positive impact on women's wage. Consequently, the wage gap between men and women at the new steady state is lower than the one before the shock; this is illustrated in Figure 9. Figure 10 shows how the shock affects female labor participation

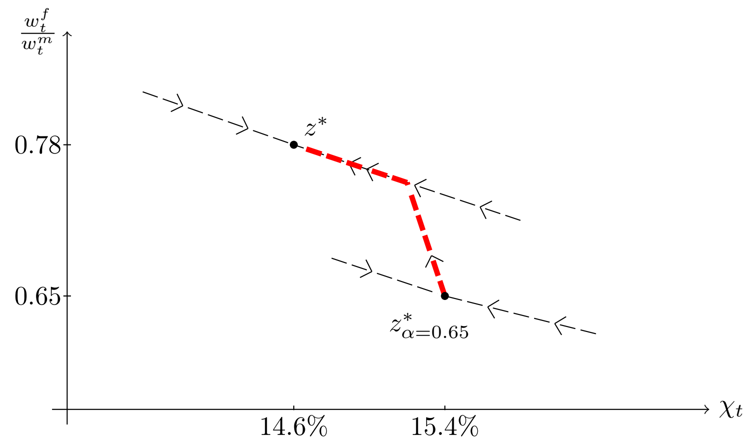

Figure 9: Effect of a decrease in α on $\frac{w_t^f}{w_t^m}$ and χ_t.

relative to men's, l; this increases at the steady state. This is explained by a decrease in the fertility of mothers, which is the consequence of an increase in their wages (implying a higher opportunity cost to have children) and an increase in their time available to work. The fluctuations are due to the opposite effects on relative labor of the decrease in the fertility of mothers (this increases l) and the increase in the proportion of households of type 3 along with the decrease in childlessness (decreases l). This increase in lifetime market participation of women is well documented in O'Neill and Polachek (1993). Our result is also coherent with the one of Jones et al. (2003) where a decrease in the gender wage gap increases the labor supplied by married women.

The same type of effects would also appear by introducing a "mommy discrimi-

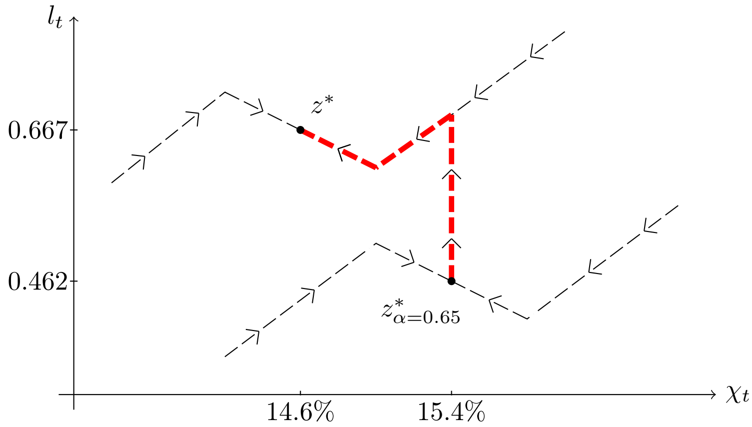

Figure 10: Effect of a decrease in α on l_t and χ_t.

nation" parameter[15] making the hourly wage of a mother lower than the hourly wage of a childless women. A decrease in this mommy discrimination parameter decreases the wage gap between mothers and non mothers and has the same effect on average fertility and childlessness (Figure 8), on the fertility of mothers (both decrease), on relative labor (Figure 10) and on the wage gap between men and women (Figure 9) than a negative shock on α. The only difference between this shock and the last one is that the wage of men increases and the wage of childless women decreases at the steady state[16].

Increase in the fixed cost of children (increase in k): An increase in k could explain the dynamics of childlessness and fertility for the cohorts born between 1930 and 1944, for whom we observe a positive relationship between fertility and childlessness. This shock mainly affects the fertility of mothers negatively. Both average fertility and childlessness are lower after the shock. The lower childlessness rate is again due to an increase in the proportion of children coming from couples of type 3 that are less likely to become childless. This means that once you pay the fixed cost, because it is bigger, you are more likely to have more children, who in turn are less likely to become childless. If we consider that the fixed cost can be interpreted as the price of a house, the increase in the real index of housing prices

[15] The amount of labor supplied by women would then become $L_t^f = L_t^{f1} + \delta L_t^{f2} + \delta L_t^{f3}$ where δ reflects the fact that the hourly wage of a married mother is lower than the one of a childless married women. The existence of this type of discrimination is confirmed in Mincer and Polachek (1974) and explained in Erosa et al. (2005) by the fact that childless women have a higher attachment to labor; consequently, they invest more time to it and become more experienced. It is also argued that a reason for this wage gap between mothers and non mothers is due to the career interruptions that women have to take each time they have a child, and that this reduction in labor supply is done at an age when the returns to labor are high.

[16] In order to simulate this shock, we need to use the utility function given in Equation 13 because it is less sensible to changes in women wages. With the linear utility function in n_t^j, what happens is that the introduction of the mommy discrimination δ pushes all women to becoming childless. In the simulations I use $\mu = 0.3$.

between 1955 and 1970 (see Skinner (1991), Figure 1) can be an explanation of the positive relationship between childlessness and fertility for cohorts of women born between 1930 and 1944. The effect on the labor market variables is negligible; the variable that is affected the most is the amount of labor supplied by women which increases since mothers have less children. Figure 11 gives an illustration of the effect of this shock on childlessness and average fertility.

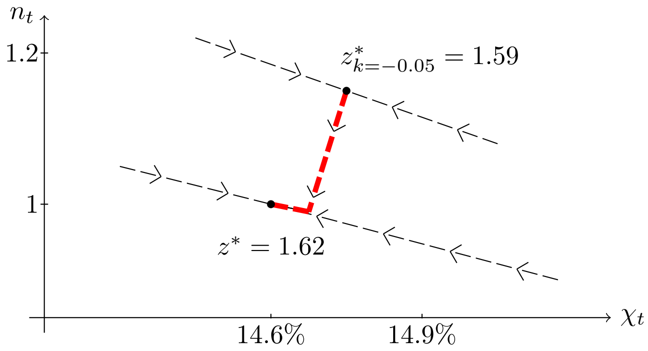

Figure 11: Effect of an increase of k on n_t and χ_t.

6 Conclusion

The aim of this work was to build a theoretical framework that could account for the fluctuations observed for childless women and to understand the economic mechanisms behind. The main results of this research are that shocks in the labor market that increase the labor opportunity of mothers, or reduce the gender wage gap, can be at the origin of the fluctuations both in childlessness and in average fertility that we have observed in the United States since the beginning of last century. The model also brings an explanation for the positive relationship between childlessness and fertility for the cohorts born during the second world war due to a possible increase in the fixed cost of becoming parents. A nice extension of the model would be to include the possibility of men and women to remain single since single women are much more likely to remain childless than married women (57.6% compared to 14.6% according to the U.S. Census Bureau, 2008).

A Tables for completed fertility and childlessness

Table A1: Childlessness rate and CEB in the United States for ever-married women born between 1826 and 1965.

Census	Age	Birth Cohort	Childlessness rate	CEB
1900	70-74	1826-1830	10.61%	5.51
1900	65-69	1831-1835	10.23%	5.54
1900	60-64	1836-1840	10.45%	5.38
1900	55-59	1841-1845	9.57%	5.35
1900	50-54	1846-1850	10.01%	5.29
1900	45-49	1851-1855	9.35%	5.25
1910	50-54	1856-1860	8.88%	5.51
1910	45-49	1861-1865	9.63%	5.32
1910	40-44	1866-1870	10.46%	4.98
1940	65-69	1871-1875	19.22%	3.30
1940	60-64	1876-1880	20.61%	2.93
1940	55-59	1881-1885	22.07%	2.73
1940	50-54	1886-1890	22.18%	2.54
1940	45-49	1891-1895	22.92%	2.33
1950	50-54	1896-1900	23.60%	2.25
1950	45-49	1901-1905	25.06%	2.07
1960	50-54	1906-1910	20.25%	2.36
1960	45-49	1911-1915	17.55%	2.41
1970	50-54	1916-1920	13.45%	2.58
1970	45-49	1921-1925	9.95%	2.84
1980	50-54	1926-1930	8.31%	3.08
1980	45-49	1931-1935	6.97%	3.21
1990	50-54	1936-1940	7.07%	2.98
1990	45-49	1941-1945	8.25%	2.56
1990	40-44	1946-1950	10.18%	2.22
1995	40-44	1951-1955	12.30%	2.09
2000	40-44	1956-1960	13.40%	2.05
2005	40-44	1961-1965	13.89%	2.02

Sources: Personal computations based on US Census data. The data for cohorts 1951 to 1965 are taken from the Table SF2 "Distribution of Women 40 to 44 Years Old by Number of Children Ever Born and Marital Status: Selected Years, 1970 to 2008" of the Census Bureau.

Table A2: Childlessness rate and CEB in Netherlands for women born between 1900 and 1959.

Birth Cohort	Childlessness rate	Completed fertility
1900-1904	23.0%	2.86
1905-1909	22.0%	2.81
1910-1914	20.0%	2.87
1915-1919	26.0%	2.96
1920-1924	15.0%	2.87
1925-1929	14.0%	2.72
1930-1934	12.0%	2.61
1935-1939	12.0%	2.38
1940-1944	12.0%	2.13
1945-1949	11.0%	1.94
1950-1954	15.0%	1.88

Sources: Data for childlessness is taken from Rowland (2007) and for completed fertility from INED, for the cohorts 1900 to 1914, completed fertility is available only once every five years, for the others, averages from single years are computed.

Table A3: Childlessness rate and CEB for women born between 1900 and 1959 in 15 OECD countries.

Country	Childlessness rate	Completed fertility
Netherlands (NLD)	18.3%	1.77
United States (USA)	14.4%	2.07
Austria (AUT)	21.1%	1.64
Norway (NOR)	12.1%	2.06
Sweden (SWE)	12.9%	1.98
Denmark (DNK)	12.7%	1.92
Slovakia (SVK)	11.1%	2.04
Portugal (PRT)	4.0%	1.82
Romania (ROU)	11.5%	1.91
Spain (ESP)	13.1%	1.59
Hungaria (HUN)	9.6%	1.97
Greece(GRE)	16.3%	1.72
Czech Republic (CZE)	7.2%	1.93
Bulgaria (BLG)	4.4%	1.83
Finland (FIN)	19.9%	1.91

Source: OECD.

B Additional figures

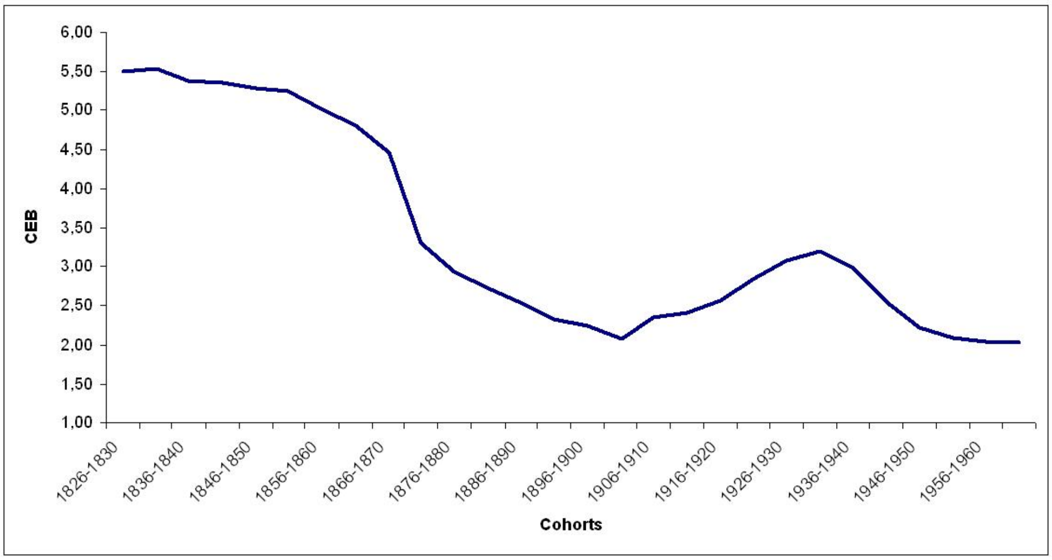

Figure 12: Completed fertility of US ever-married women by cohorts.

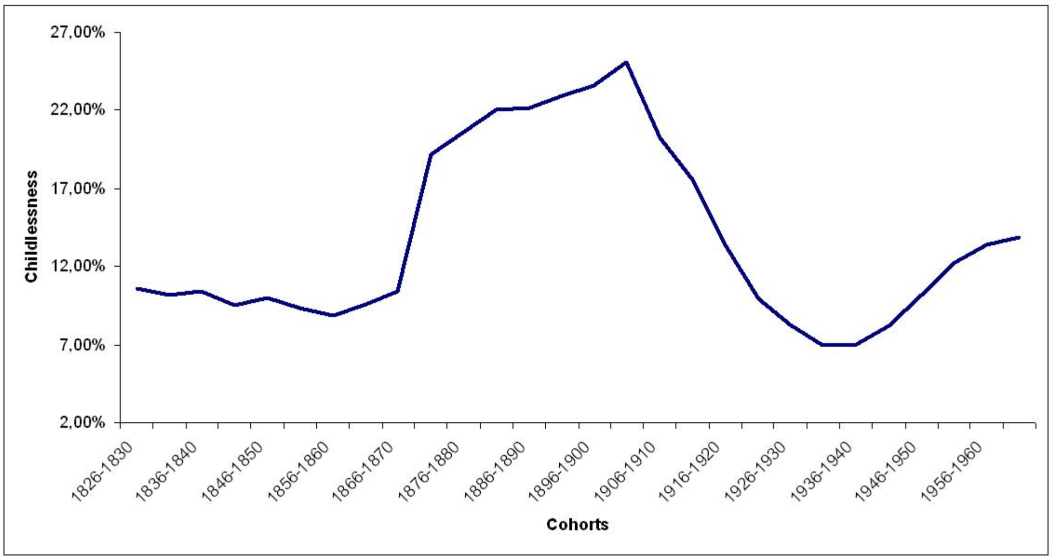

Figure 13: Percentage of childless among ever-married women by cohorts in the US.

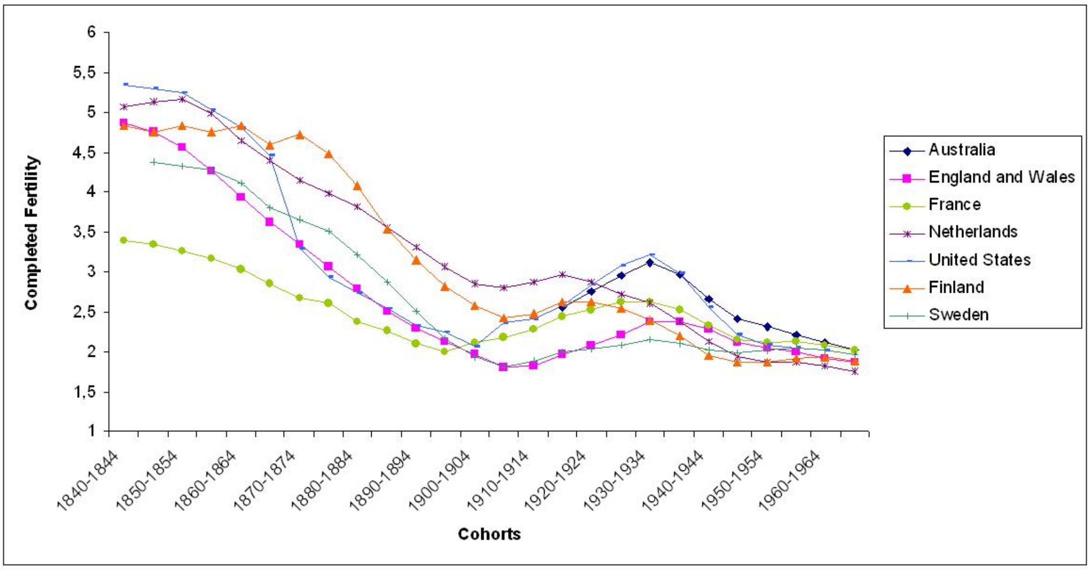

Figure 14: Completed fertility by cohorts.

Source: U.S. Census for the years 1900, 1910, 1940, 1950, 1960, 1970, 1980 and 1990 for the United States and INED for the other countries. Note: for the US, completed fertility is for ever-married women whereas for the rest of the countries all women are considered.

C Proof of Proposition 3.1

The interior solution is optimal if the utility of having children is higher than the one of remaining childless, that is, if the following condition is satisfied,

$$\ln\left(\frac{\theta w^f}{\gamma^j}\right) + \gamma^j\left(\frac{w^m + w^f - k}{\theta w^f} - \frac{1}{\gamma^j}\right) \geq \ln\left(w^m + w^f\right)$$

that can be rewritten in the following way,

$$\ln\left(\frac{\theta w^f}{\gamma^j\left(w^m + w^f\right)}\right) \geq 1 - \gamma^j\frac{w^m + w^f - k}{\theta w^f}$$

Denoting by v and z the following functions,

$$v(\gamma^j) = \ln\left(\frac{\theta w^f}{\gamma^j\left(w^m + w^f\right)}\right)$$

and

$$z(\gamma^j) = 1 - \frac{\gamma^j(w^m + w^f - k)}{\theta w^f}$$

the interior solution is optimal if $v(\gamma^j) \geq z(\gamma^j)$. Studying the function $v(\gamma^j)$, we have that $v'(\gamma^j) < 0$ and $v''(\gamma^j) > 0$, so that the function $v(\gamma^j)$ is decreasing and convex. The limits are the following,

$$\lim_{\gamma^j \to 0^+} v(\gamma^j) = +\infty$$

$$\lim_{\gamma^j \to +\infty} v(\gamma^j) = -\infty$$

and

$$v(\gamma^j) = 0 \Leftrightarrow \gamma^j = \frac{\theta w^f}{w^m + w^f}$$

For the function $z(\gamma^j)$, we have that $z'(\gamma^j) < 0$ and $z''(\gamma^j) = 0$, so that $z(\gamma^j)$ is linearly decreasing. We then have the following,

$$z(0) = 1$$

$$\lim_{\gamma^j \to +\infty} z(\gamma^j) = -\infty$$

and

$$z(\gamma^j) = 0 \Leftrightarrow \gamma^j = \frac{\theta w^f}{w^m + w^f - k}$$

Since $v(\gamma^j)$ is decreasing and convex and $z(\gamma^j)$ is decreasing but linear, we have that,

$$\lim_{\gamma^j \to +\infty} (v(\gamma^j) - z(\gamma^j)) > 0$$

so that for large values of γ^j, the interior solution is optimal. Note that at the value $\gamma^j = \frac{\theta w^f}{w^m + w^f - k}$, which corresponds to $n^* = 0$, the corner solution is optimal, since,

$$\begin{aligned} v\left(\frac{\theta w^f}{w^m + w^f - k}\right) - z\left(\frac{\theta w^f}{w^m + w^f - k}\right) &= \ln\left(\frac{\theta w^f}{\frac{\theta w^f}{w^m + w^f - k}(w^m + w^f)}\right) - 0 \\ &= \ln\left(\frac{w^m + w^f - k}{w^m + w^f}\right) \\ &< 0 \end{aligned}$$

Consequently, we know that the two functions, $v(\gamma^j)$ and $z(\gamma^j)$, will intersect twice, once before the value $\gamma^j = \frac{\theta w^f}{w^m + w^f - k}$ and once after. For $\gamma^j < \frac{\theta w^f}{w^m + w^f - k}$, the constraint int $n_t^j \geq 0$ is not respected, so we only need to consider the values for $\gamma^j \geq \frac{\theta w^f}{w^m + w^f - k}$. This allows us to conclude that there will be a value of γ^*, where $v(\gamma^*) = z(\gamma^*)$, where couples are indifferent between having children or being childless.

Effect of a change in wages on γ^*: Applying the implicit function theorem to the function Ξ defined as,

$$\Xi(\gamma, w^m, w^f, k) = v(\gamma, w^m, w^f, k) - z(\gamma, w^m, w^f, k)$$

we can check that $\frac{\delta\gamma^*}{\delta w^m} < 0$ meaning that an increase in the wage of men reduces the critical level γ^*. The relationship between γ^* and w^f is not clear and γ^* increases with the fixed cost k.

D Another utility function

The following utility function could also be used,

$$U_t^j(c_t^j, n_t^j) = \ln\left(c_t^j\right) + \gamma^j \ln\left(\mu + n_t^j\right) \tag{13}$$

where μ can be interpreted as a substitution parameter between consumption and fertility for the couple. The first order conditions for n_t^j and c_t^j are the following ones:

$$n_t^j = \begin{cases} \frac{1}{\theta} & \text{if } w^f \leq \frac{\gamma^j(w^m-k)}{1+\theta\mu} \\ \frac{\gamma^j}{1+\gamma^j}\frac{w^m+w^f-k}{\theta w^f} - \frac{\mu}{1+\gamma^j} & \text{if } \frac{\gamma^j(w^m-k)}{1+\theta\mu} < w^f < \frac{w^m-k}{\frac{\theta\mu}{\gamma^j}-1} \\ 0 & \text{if } w^f \geq \frac{w^m-k}{\frac{\theta\mu}{\gamma^j}-1} \end{cases}$$

and

$$c_t^j = \begin{cases} w^m - k & \text{if } w^f \leq \frac{\gamma^j(w^m-k)}{1+\theta\mu} \\ \frac{w^m+w^f(1+\theta\mu)-k}{1+\gamma^j} & \text{if } \frac{\gamma^j(w^m-k)}{1+\theta\mu} < w^f < \frac{w^m-k}{\frac{\theta\mu}{\gamma^j}-1} \\ w^m + w^f & \text{if } w^f \geq \frac{w^m-k}{\frac{\theta\mu}{\gamma^j}-1} \end{cases}$$

The interior solution is optimal if the following inequation is satisfied:

$$\ln\left(\frac{w^m + w^f(1+\theta\mu) - k}{1+\gamma^j}\right) + \gamma^j \ln\left(\frac{\gamma^j}{1+\gamma^j}\frac{w^m + w^f - k}{\theta w^f} + \frac{\mu\gamma^j}{1+\gamma^j}\right) \geq \ln\left(w^m + w^f\right) + \gamma^j \ln \mu$$

In order to go further with the analytical results, I chose the utility function given in Equation (1) and not the one proposed in this Appendix. This does not change the qualitative aspect of the simulations, and we would have the same conclusions with one utility function or the other. However, the linearity of the utility function in n_t^j makes it more sensitive to changes in the female wages than what it would be with a utility function such as Equation (13).

E Relaxing random matching assumption

The random matching assumption might seem unrealistic and too simplificative of the marriage market, because high rates of homogamy are found for some social groups such as French aristocrats, individuals of a particular religion or among educated and non educated individuals (see Bisin and Verdier (2000)). Random matching would be a wrong way to model marriage if individuals would differ in one of these observable traits, since it would clearly underestimate homogamous marriages. However, in our case, we have heterogeneity in taste for children, which is not an observable characteristic (unlike the social class, religion, ethnicity or education). To my knowledge, there is no evidence that individuals with high or low taste for children have a tendency to match together. Moreover, homogamous marriages mainly arise because of a certain value or characteristic the parents want to transmit to their children. I do not know of any study showing that the taste for children is a characteristic that parents want to transmit (it cannot be the case of childlessness of course) and consequently a characteristic that segregates the marriage market.

However, since *"preferences are more likely to be positively than negatively sorted"*,[17] I look at what would change by introducing some assortative matching between individuals of the same type. This means that individuals with the same tastes for children will be more likely to be together than in the random matching framework. Letting λ denote the degree of "assortativeness", the proportions of each type of couple, at time t, are the following:

1. Type 1: $\left(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2(1-\lambda)+\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\lambda$
2. Type 2: $\frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}(1-\lambda)$
3. Type 3: $\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2(1-\lambda)+\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\lambda$

It is easy to notice that $\lambda = 0$ corresponds to the random matching case and $\lambda = 1$ means that there are no mixed couples, so that individuals from different types do not form a couple (perfect assortative matching case). We can rewrite the proportions in terms of z_t as follows:

1. Type 1: $\left(\frac{1}{1+z_t}\right)^2(1-\lambda)+\frac{1}{1+z_t}\lambda$
2. Type 2: $\frac{2z_t}{(1+z_t)^2}(1-\lambda)$
3. Type 3: $\left(\frac{z_t}{1+z_t}\right)^2(1-\lambda)+\frac{z_t}{1+z_t}\lambda$

[17] Becker (1993), pages 123-124.

The dynamics of z_t can then be rewritten such as,

$$z_{t+1} = \frac{a\overline{\overline{n}}(z_t + \lambda) + 2b\overline{n}(1-\lambda)}{(1-a)\overline{\overline{n}}(z_t + \lambda) + 2(1-b)\overline{n}(1-\lambda)} \equiv \phi_a(z_t) \tag{14}$$

The case of perfect assortative matching, $\lambda = 1$ implies that we do not have any dynamics: we are always at the steady state, equal to $\frac{a}{1-a}$. The first derivative of $\phi_a(z_t)$ is given by,

$$\phi_a'(z_t) = \frac{2\overline{\overline{n}}(1-\lambda)\overline{n}(a-b)}{\big((1-a)\overline{\overline{n}}(z_t+\lambda) + 2(1-b)\overline{n}(1-\lambda)\big)^2} \geq 0 \iff a \geq b$$

and the second derivative by,

$$\phi_a''(z_t) = \frac{4\overline{\overline{n}}^2(1-\lambda)\overline{n}(a-b)(1-a)}{\big((1-a)\overline{\overline{n}}(z_t+\lambda) + 2(1-b)\overline{n}(1-\lambda)\big)^3} \leq 0 \iff a \geq b$$

We also have that,

$$\phi_a(0) = \frac{a\overline{\overline{n}}\lambda + 2b\overline{n}(1-\lambda)}{(1-a)\overline{\overline{n}}\lambda + 2(1-b)\overline{n}(1-\lambda)} \geq \frac{b}{1-b} \iff a \geq b$$

and

$$\lim_{z_t \to \infty} \phi_a(z_t) = \frac{a}{1-a} > 0$$

This means that we will also have a unique positive steady state in the case of assortative matching. Convergence will be faster and the same type of dynamics will arise as in the random matching case.

F Exogenous probabilities: cases $a = b$ and $a < b$

Case $a = b$: We have that,

$$\phi(z_t) = \frac{a}{1-a}$$

so that $\phi'(z_t) = 0$. This says that the function ϕ is a constant and that the steady state is reached in one period: if z_0 is lower or higher than z^*, then in period one we will be at the steady state, which is globally stable as before and depends only in the value of a.

Proposition F.1. *If $a = b$, the steady state is reached in one period.*

Case $a < b$: This is when the probability for a child to have a high taste for children, is higher for children of couples of type 2 than for children having parents with the highest willingness. We have that, $\phi'(z_t) < 0$ and $\phi''(z_t) > 0$. To analyze the stability of z^* in this case, I will proceed in two steps: first, I will compute the value $\bar{z}$ for which we have $\phi'(\bar{z}) = -1$ and then compare it to the steady state value z^* that we already computed. If $z^* > \bar{z}$ then $0 > \phi'(z^*) > \phi'(\bar{z}) = -1$ and consequently, z^* is locally stable. If $z^* < \bar{z}$ then $\phi'(z^*) < -1$ and z^* is unstable. If $z^* = \bar{z}$ then z^* is non-hyperbolic since $\phi'(z^*) = \phi'(\bar{z}) = -1$.

$$\phi'(\bar{z}) = -1$$
$$\Leftrightarrow \bar{z} = \frac{-2(1-b)\overline{n} + \sqrt{2\overline{n}(b-a)\overline{\overline{n}}}}{(1-a)\overline{\overline{n}}}$$

Now, comparing this with z^*, we have that,

$$\begin{aligned} z^* - \bar{z} = & \frac{-\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right) + \sqrt{\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right)^2 + 2(1-a)\overline{n}b\overline{\overline{n}}}}{(1-a)\overline{\overline{n}}} \\ & - \frac{-2(1-b)\overline{n} + \sqrt{2\overline{n}(b-a)\overline{\overline{n}}}}{(1-a)\overline{\overline{n}}} \end{aligned}$$

giving,

$$z^* - \bar{z} = \frac{\sqrt{\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right)^2 + 2(1-a)\overline{n}b\overline{\overline{n}}} - \sqrt{2\overline{n}(b-a)\overline{\overline{n}}} + \frac{a}{2}\overline{\overline{n}} + (1-b)\overline{n}}{(1-a)\overline{\overline{n}}}$$

and by looking at the squared roots of the numerator, we can easily conclude that,

$$z^* > \bar{z}$$

Since the function $\phi'(z_t)$ is increasing when $a < b$ we can say that,

$$0 > \phi'(z^*) > \phi'(\bar{z}) = -1$$

consequently, the steady state z^* is locally stable.

Proposition F.2. *If $a < b$, the dynamics are oscillatory and z^* is locally stable.*

The relationship $a < b$ means that children of small families are more likely to have a high taste for children than the ones born in bigger families. This might seem a little unrealistic, but we could argue that if we consider a family with only one child, this child might feel lonely during his childhood and therefore will not want his own children to feel the same way, so that he will want more children than his parents did. The opposite could happen in big families where children get fed up with noise and disorder. The intuition behind the oscillations can be due to the following mechanism: suppose that we start from a low level $z_0 < z^*$; as before we will have a higher proportion of children coming from the mixed couples, but now, those are the most likely to have a high taste for children, consequently in

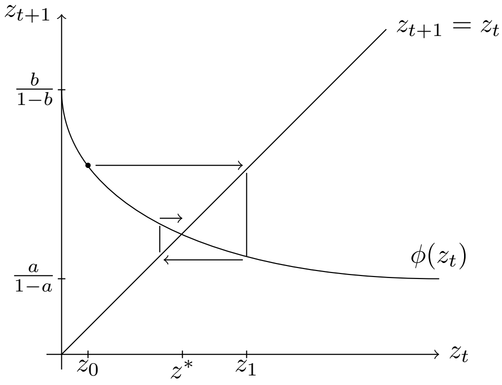

Figure 15: Oscillatory dynamics in the case $a < b$

period 1 there will be many individuals with high taste for children and z_1 will be high. In period 2, the opposite will happen; a higher proportion of individuals coming from the third type of marriage, who are less likely to have a high taste for children, then z_2 will be low, and this will continue until reaching the steady state.

G Dynamics with endogenous probabilities

Here I enlarge the model assuming that the probability of having a child either with high taste for children when adult, $\overline{\gamma}$, or low taste, $\underline{\gamma}$, depends both on the willingness for children of the parents and on the average fertility of population. This is in line with Bisin and Verdier (2001) where the traits of children depend both on the preferences of their parents and on the social environment. An empirical justification for this framework is given in Fernández and Fogli (2006) where the authors show that both family experience and cultural heritage are two determinant factors of the fertility choice. Accordingly, we consider the following probability functions,

$$a_t = (\overline{\gamma})^{\tau}(n_t)^{\eta}$$

and

$$b_t = \left(\frac{\overline{\gamma} + \underline{\gamma}}{2}\right)^{\tau}(n_t)^{\eta}$$

where $\tau \in [-1, 1]$ determines the weight of parental willingness for children on the taste of their own children and $\eta \in [-1, 1]$ can be interpreted as an externality of the average fertility influencing the taste for children, in other words, how the fertility behavior of one generation affects the taste for children of the next one. In Fernández and Fogli (2006) they show that "women whose parents were born in countries where women had more children, tend to have more children themselves", supporting the idea that $\eta > 0$, and "women from larger families tend to have more children", supporting $\tau > 0$, this last relationship between the taste of parents and the one of children is also sustained in Ben-Porath (1975). In Berent (1953) the author tests the hypothesis that family size runs through generations (which corresponds to $\tau > 0$ in our case); this hypothesis is verified in the population studied (married women in Great Britain), indeed, couples coming from higher families had themselves a higher fertility in average (Table 1 in Berent (1953)). In Rowland (2007), it is also argued that, in Australia, the birth cohorts that had the lowest average family size also had the highest childlessness rate.

Replacing average fertility in a_t and b_t and then introducing these probability functions into the difference equation (8), we can obtain with some simple arrangements the following expression for the dynamics of z_t,

$$z_{t+1} = \frac{\left(\left(\frac{z_t}{z_t+1}\right)^2 \overline{\overline{n}} + \frac{2z_t}{(z_t+1)^2}\overline{n}\right)^\eta \left((\overline{\gamma})^\tau \overline{\overline{n}} z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n}\right)}{\overline{\overline{n}} z_t + 2\overline{n} - \left(\left(\frac{z_t}{z_t+1}\right)^2 \overline{\overline{n}} + \frac{2z_t}{(z_t+1)^2}\overline{n}\right)^\eta \left((\overline{\gamma})^\tau \overline{\overline{n}} z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n}\right)} \equiv \Phi(z_t) \tag{15}$$

This expression does not provide direct analytical results, but we can still take some intuitions from it by studying five cases: $\eta = 1$, $\eta = \frac{1}{2}$, $\eta = 0$, $\eta = -\frac{1}{2}$ and $\eta = -1$.

Case $\eta = 1$: We can rewrite the expression given in equation (15) in the following way,

$$z_{t+1} = \frac{(\overline{\gamma})^\tau \overline{\overline{n}} z_t^2 + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n} z_t}{(z_t+1)^2 - \left((\overline{\gamma})^\tau \overline{\overline{n}} z_t^2 + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n} z_t\right)} \equiv \Phi_{\eta=1}(z_t)$$

This dynamic of z_t has two steady states that can be computed analytically. One is the trivial solution, $z^* = 0$, and the other one is the following,

$$z^* = \frac{-1 + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n}}{1 - (\overline{\gamma})^\tau \overline{\overline{n}}}$$

for which the sign is unknown but it is likely to be negative.

A numerical analysis of this case tells us that the only steady state that seems to exist is the trivial one. For high values of τ, this steady state is stable (implying that $n^* = 0$ and $\chi^* = 1$) and it becomes unstable for lower values of τ. This means that when people are very influenced by other people's behavior, then everyone becomes childless.

Case $\eta = \frac{1}{2}$: We have,

$$z_{t+1} = \frac{\left(z_t^2\overline{\overline{n}} + 2z_t\overline{n}\right)^{\frac{1}{2}}\left(\left(\overline{\gamma}\right)^{\tau}\overline{\overline{n}}z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{n}\right)}{\left(z_t+1\right)\left(\overline{\overline{n}}z_t + 2\overline{n}\right) - \left(z_t^2\overline{\overline{n}} + 2z_t\overline{n}\right)^{\frac{1}{2}}\left(\left(\overline{\gamma}\right)^{\tau}\overline{\overline{n}}z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{n}\right)} \equiv \Phi_{\eta=\frac{1}{2}}(z_t)$$

Other than the trivial steady state, we can have two other steady states that are the roots of the following second order linear equation,

$$\overline{\overline{n}}\left(1 - \left(\overline{\gamma}\right)^{2\tau}\overline{\overline{n}}\right)z^2 + 2\overline{n}\left(1 - 2\left(\overline{\gamma}\right)^{\tau}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{\overline{n}}\right)z - 4\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{2\tau}\overline{n}^2 = 0$$

which discriminant is,

$$\Delta = 4\overline{n}^2\left(1 + 4\overline{\overline{n}}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\left[\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau} - \left(\overline{\gamma}\right)^{\tau}\right]\right)$$

If $\Delta > 0$, the two real roots are given by the following expressions:

$$z_1^* = \frac{-\overline{n}\left(1 - 2\left(\overline{\gamma}\right)^{\tau}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{\overline{n}}\right) - \overline{n}\sqrt{1 + 4\overline{\overline{n}}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\left[\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau} - \left(\overline{\gamma}\right)^{\tau}\right]}}{\overline{\overline{n}}\left(1 - \left(\overline{\gamma}\right)^{2\tau}\overline{\overline{n}}\right)}$$

and

$$z_2^* = \frac{-\overline{n}\left(1 - 2\left(\overline{\gamma}\right)^{\tau}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{\overline{n}}\right) + \overline{n}\sqrt{1 + 4\overline{\overline{n}}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\left[\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau} - \left(\overline{\gamma}\right)^{\tau}\right]}}{\overline{\overline{n}}\left(1 - \left(\overline{\gamma}\right)^{2\tau}\overline{\overline{n}}\right)}$$

Stability at $z^* = 0$: We can study the stability at the trivial steady state by looking at the first derivative of $\Phi_{\eta=\frac{1}{2}}(z_t)$ at $z_t = 0$, even though it is not defined for $z_t = 0$. We define $\Phi'_{\eta=\frac{1}{2}}(0)$ the following limit:

$$\Phi'\left(0^+\right) \equiv \lim_{z_t \to 0^+} \Phi'(z_t)$$

Using the definition of the derivative at one point, we have that,

$$\begin{aligned}\Phi'\left(0^+\right) &= \lim_{z \to 0^+} \frac{\Phi(z) - \Phi(0)}{z - 0} \\ &= \lim_{z \to 0^+} \frac{\Phi(z)}{z}\end{aligned}$$

and studying the function $\frac{\Phi(z)}{z}$, we have that,

$$\Phi'(0^+) = \lim_{z \to 0^+} \frac{\Phi(z)}{z} = +\infty$$

Consequently, the trivial steady state is locally unstable.

Numerical simulations tell us that for some positive values of τ, the dynamics are monotonic and converge to a positive and stable steady state. This can be interpreted in the same way as the case $a > b$ with exogenous probabilities. For low and negative values of τ, the only steady state is the trivial one.

Case $\eta = 0$: The dynamics are given by the following expression,

$$z_{t+1} = \frac{\overline{\overline{n}} z_t (\overline{\gamma})^{\tau} + 2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}}{\overline{\overline{n}} z_t \left(1-(\overline{\gamma})^{\tau}\right) + 2\overline{n}\left(1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right)} \equiv \Phi_{\eta=0}(z_t)$$

Note that this function is negative if $\tau < 0$ because a_t and b_t become higher than 1 and the denominator becomes negative, consequently, we can only study this function for $\tau > 0$ (the function is not defined for $\tau = 0$). The trivial steady state is no longer present but there are two real steady states given by,

$$z_1^* = \frac{-\left(2\overline{n}\left[1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right] - \overline{\overline{n}}\,(\overline{\gamma})^{\tau}\right) - \sqrt{\Delta}}{2\overline{\overline{n}}\,(1-(\overline{\gamma}))}$$

and

$$z_2^* = \frac{-\left(2\overline{n}\left[1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right] - \overline{\overline{n}}\,(\overline{\gamma})^{\tau}\right) + \sqrt{\Delta}}{2\overline{\overline{n}}\,(1-(\overline{\gamma}))}$$

where,

$$\Delta = \left(2\overline{n}\left[1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right] - \overline{\overline{n}}\,(\overline{\gamma})^{\tau}\right)^2 + 8\overline{n}\left(1-(\overline{\gamma})^{\tau}\right)\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{\overline{n}}$$

The first derivative of $\Phi_{\eta=0}(z_t)$ is given by,

$$\Phi'_{\eta=0}(z_t) = \frac{2\overline{n}\left[(\overline{\gamma})^{\tau} - \left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]\overline{\overline{n}}}{\left[\overline{\overline{n}} z_t \left(1-(\overline{\gamma})^{\tau}\right) + 2\overline{n}\left(1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right)\right]^2} > 0$$

and the second derivative by,

$$\Phi''_{\eta=0}(z_t) = \frac{-4\overline{n}\left[(\overline{\gamma})^{\tau} - \left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]\overline{\overline{n}}^2\left(1-(\overline{\gamma})^{\tau}\right)}{\left[\overline{\overline{n}} z_t \left(1-(\overline{\gamma})^{\tau}\right) + 2\overline{n}\left(1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right)\right]^3} < 0$$

so that the dynamics of z_t will converge to the positive steady state, and this will be globally stable.

Here the probabilities are only affected by the preferences of the parents; this case is similar to the last model with exogenous probabilities since a_t and b_t remain constant over time; furthermore, which of these is bigger depends on the parameter τ: $\tau > 0$ will lead to $a_t > b_t$ and $\tau < 0$ will lead to $a_t < b_t$. I show in Appendix E that the dynamics of z_t will converge monotonically to a positive steady state, and this will be globally stable. The intuition is the same as the one given for exogenous probabilities.

Case $\eta = -\frac{1}{2}$ and $\eta = -1$: The dynamics are given by the expressions,

$$z_{t+1} = \frac{(1+z_t)\left(\overline{\overline{n}}z_t^2 + 2z_t\overline{n}\right)\left(\left(\overline{\gamma}\right)^{\tau}\overline{\overline{n}}z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{n}\right)}{\overline{\overline{n}}z_t + 2\overline{n} - (1+z_t)\left(\overline{\overline{n}}z_t^2 + 2z_t\overline{n}\right)\left(\left(\overline{\gamma}\right)^{\tau}\overline{\overline{n}}z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{n}\right)} \equiv \Phi_{\eta=-\frac{1}{2}}(z_t)$$

and

$$z_{t+1} = \frac{z_t\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau} + 2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}}{\frac{z_t\left(z_t\overline{\overline{n}}+2\overline{n}\right)^2}{(1+z_t)^2} - \left[z_t\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau} + 2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]} \equiv \Phi_{\eta=-1}(z_t)$$

with,

$$\Phi'_{\eta=-1}(z_t) = \frac{\frac{z_t\overline{\overline{n}}+2\overline{n}}{1+z_t}\left[z_t^2\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau}\frac{2\left(\overline{n}-\overline{\overline{n}}\right)-z_t\overline{\overline{n}}}{1+z_t} - 2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\left(\frac{z_t\overline{\overline{n}}+2\overline{n}}{1+z_t} + 2z_t\overline{\overline{n}}\right)\right]}{\left(\frac{z_t\left(z_t\overline{\overline{n}}+2\overline{n}\right)^2}{(1+z_t)^2} - \left[z_t\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau} + 2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]\right)^2} < 0$$

which is negative because $\frac{2\left(\overline{n}-\overline{\overline{n}}\right)-z_t\overline{\overline{n}}}{1+z_t} < 0$.

In these two cases, the dynamics are oscillatory and converge to a unique positive steady state level that exists for values of τ not too small; otherwise, there is no steady state. This case differs from the previous results because we can have oscillations even if $a_t > b_t$. When the fertility behavior of the past generation negatively affects the tastes over fertility of the generation that follows, children in large families get fed up with children, although they originally may have a high taste for children. Starting with a high level of z_0, we would also have a high fertility level n_0, meaning that the adults at $t = 1$ would feel as being too many, and would consequently have a lower willingness to procreate, which would decrease their fertility n_1.

Brief summary of the results:

- If $\tau > 0$ and $\eta > 0$: monotonic dynamics with a stable steady state.
- If $\tau > 0$ and $\eta < 0$: oscillatory dynamics with a stable steady state.

- If $\tau < 0$: only the trivial steady state and unstable ($\eta > 0$), no steady state ($\eta < 0$ close to zero), or a stable steady state with oscillatory dynamics ($\eta < 0$ and τ not too small).

This last Appendix tells us that the dynamics of childlessness can be explained by the dynamics of preferences when the taste for children reacts negatively to the fertility rate of the past generation. This last hypothesis would, however, contradict one of the results of Fernández and Fogli (2006).

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